A Low-Complexity PTS Scheme for PAPR Reduction in MIMO-OFDM Systems

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Abstract— **In this paper, we consider the advantages of multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) system with space-frequency block coding (SFBC) is that the signals transmitted on different antennas might exhibit a prohibitively large peak-to-average power ratio (PAPR). Partial transmit sequence (PTS) provides attractive PAPR reduction performance in OFDM or MIMO-OFDM. But unfortunately, PTS is straightforwardly applied to MIMO-OFDM systems, usually denoted as traditional ordinary PTS, which results in large computational complexity. It proposes the application of the Firefly algorithm (FA) to solve the problem. Computer simulation results show that the proposed FA obtains the desired PAPR reduction with low computational complexity dramatically and achieve better PAPR reduction performance compared to ordinary PTS. In addition, the proposed PTS schemes can achieve a reduced PAPR performance and a bit error rate performance close to that of SFBC MIMO-OFDM systems, but at much lower computational cost**

Keywords—MIMO, OFDM, PAPR, Firefly algorithm, PTS

Introduction

To achieve the high data rates required by 5G, many technologies have been proposed [1-2]. The combination of Multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) techniques, namely, MIMO-OFDM, has become one of the most attractive candidates for broadband wireless communications, such as wireless metropolitan area networks (WMANs) and 3GPP Long Term Evolution (LTE) systems [3-4]. The main advantages of using MIMO-OFDM system over single carrier system include resistant to impulse interference, capability of handling very strong echoes, high power spectral efficiency, robustness to channel fading, lesser nonlinear distortion, use of small guard intervals and uniform average spectral density [5-6].The major disadvantage of MIMO-OFDM systems is the high peak-to-average power ratio (PAPR) at the transmitter's output signal on different antennas. High PAPR for MIMO-OFDM system is still a demanding area and difficult issue as the occurrence of high peaks in the transmitted OFDM signal results in the degradation of the system performance due to various nonlinear effects like spectral spreading, intermodulation and signal constellation that exist inherently in power amplifiers Recently, many studies have been proposed for MIMO-OFDM systems PAPR reduction methods [6-11]. Among these, cross-antenna rotation and inversion, partial transmit sequence (PTS) and selected mapping (SLM) all exhibit significant performance by exploiting the additional degrees of freedom introduced by multiple antennas. The basic idea of the conventional PTS (C-PTS) [9-10] scheme is to generate several alternative signals by multiplying the original data block with different phase rotation sequences. Then, the time-domain signal with the minimum PAPR is selected as the transmitted signal. However, for the C-PTS scheme, extra bits should be reserved for the transmission of the phase rotation sequences as the side information (SI) to recover the original data at the receiver, resulting in the decrease of the data rate. Although some PTS schemes have been proposed for the OFDM systems, no blind PTS schemes have been proposed for the Alamouti MIMO-OFDM systems [12]. Unlike to space-time block codes (STBC) MIMO-OFDM[7],where independent OFDM sequences are transmitted over different antennas at certain symbol period, in space-frequency block codes (SFBC) MIMO-OFDM [8-9] systems the transmitted sequences are dependent due to the use of space-frequency code, so that the cross-antenna rotation of cannot be employed. Therefore, we propose a method of SFBC which has gained much attention as an effective transmit diversity technique to provide reliable transmission with high peak data rates.

Partial transmit sequence (PTS) is attractive as it can obtain good PAPR property by modifying OFDM signals without any distortion. But unfortunately, PTS is straightforwardly applied to MIMO-OFDM systems, usually denoted as ordinary PTS (oPTS) [13-14], which results in large computational complexity. In PTS, alternate optimization and spatial subblock circular permutation are combined, where the use of alternate optimization results in reducing computational complexity and the use of spatial subblock circular permutation across all the transmitting antennas is able to increase the number of candidate sequences, which equivalently improves the performance for PAPR reduction. Cooperative PTS (co-PTS) method presented in [14] uses spatial subblock circular permutations across all the transmitting antennas for the odd subblocks to increase the number of candidate signals, and then uses alternate optimization to find the signal with the lowest PAPR value. In the proposed technique, a sub-optimization search is performed to find the phase factors. Further motivated by the observation above, the firefly algorithm (FA) [15-17] is a recent nature-inspired optimization algorithm that simulates the flashing pattern and behavioral characteristics of fireflies. By using an iterative procedure, FA-based PTS scheme can obtain a significant PAPR reduction as well as an improved low computational complexity performance simultaneously.

The outline of this paper is organized as follows. Section 2 briefly reviews the high PAPR problem and formulates the PTS based MIMO-OFDM system. The proposed method is described in Sec. 3. The performance comparisons of various methods are presented in Sec. 4. Finally, conclusions are made in Section 5. Information about final paper submission is available from the conference website.

1. SYSTEM MODEL DESCRIPTION

An easy way to comply with the conference paper formatting requirements is to use this document as a template and simply type your text into it. For simplicity we consider a SFBC MIMO-OFDM system that employs Alamouti scheme [12], although extension to more than two transmit antennas is possible. Consider an SFBC MIMO-OFDM system that employs Alamouti scheme although extension to more than two transmits antennas is possible [9, 12]. This system uses N subcarriers, T transmitting and R receiving antennas. Obtaining an improved approximation of the true PAPR in the discrete-time case requires the candidate signals to be over-sampled. In SFBC MIMO-OFDM systems with two transmitting antennas, the input data block symbol vector $\mathbf{X}=\left[X_0, X_1, X_2, \cdots, X_{N-1}\right]$ is encoded with space-frequency encoder into two vectors $\mathbf{X}_1$ and $\mathbf{X}_2$, as

$$\mathbf{X}_1=\left[\; X_0,\;\; -X_1^*,\cdots,\;\; X_{N-2},\;\; -X^*_{N-1} \;\right] \quad (1a)$$

$$\mathbf{X}_2=\left[\; X_1,\;\; -X_0^*,\cdots,\;\; X_{N-1},\;\; -X^*_{N-2} \;\right] \quad (1b)$$

respectively, where N is the number of subcarriers and $(\cdot)^*$ denotes the complex conjugate operation [13], which are fed to the IFFT Cyclic Prefix (CP) blocks and sent simultaneously from antennas T_{X_1} and T_{X_2}, respectively. The demodulated signal at the receiver is then given by

$$\mathbf{Y}=\Re_1\mathbf{X}_1+\Re_2\mathbf{X}_2+\mathbf{Z}, \quad (2)$$

where $\Re_1$ and $\Re_2$ represent diagonal matrices whose elements are the FFTs of the corresponding channel impulse responses $\mathbf{h}_1$ and $\mathbf{h}_2$. Vector $\mathbf{Z}$ denotes the FFT of the channel noise. Obviously, the SFBC-OFDM scheme also suffers from large PAPR, which has to be reduced. In this case cross-antenna rotation proposed in [9-11] cannot be applied to vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ since it would destroy the structure of Alamouti (or any other) space-frequency block code. The data vector $\mathbf{x}_i$ is, thus, given by

$$\mathbf{x}_i=\left[x_{i,0}, x_{i,1}, \cdots, x_{i,n}, \cdots, x_{i,(N-1)})\right]^T \quad (3)$$

where $(\cdot)^T$ denotes the matrix transpose. Since OFDM signals are modulated independently in each subcarrier, the combined OFDM signals are likely to have large peak powers at certain instances. When calculating PAPR using discrete sampled signals, we cannot find the accurate PAPR because the true peak of continuous-time OFDM signal may be missed in the Nyquist sampling. So, we use 4 times over-sampling to improve accuracy of discrete PAPR. The peak power increases as the number of subcarriers increases. Therefore, in the discrete-time case, the

PAPR of the transmitted signals of each antenna is defined as

$$PAPR(x) = 10\log_{10}\frac{\max\limits_{0\le n\le LN-1}|x_n|^2}{E\left[|x_n|^2\right]} \quad , \qquad (4)$$

where the numerator represents the maximum envelope power and the denominator represents the average power Since the transmitted signals of the two antennas correlate in SFBC MIMO-OFDM systems, the PAPR should be considered jointly and the signal pair with the lowest PAPR is selected for transmission. Therefore, the PAPR of the SFBC MIMO-OFDM [9-11] system is defined as

$$PAPR(\mathbf{x}) = \max_{i=1,2}\left\{PAPR(\mathbf{x}_i)\right\} \qquad (5)$$

where $\mathrm{x} = [\mathrm{x}_1\ \mathrm{x}_2\ ]$. Usually, Complementary Cumulative Distribution Function (CCDF) is used to show performance measure for PAPR reduction techniques. It calculates the probability that the PAPR of a data block exceeds a given threshold $PAPR_0$ and computed by Monte Carlo Simulation. The CCDF of the PAPR of N symbols of a data block with Nyquist rate sampling is defined as (3).

Partial transmit sequence (PTS) is individually applied to each transmitting antenna in a straightforward manner, called ordinary PTS (oPTS). As it is shown in [11], at each antenna, the frequency domain sequence of OFDM systems with D subcarriers is firstly partitioned into M disjoint subblocks. In the PTS method, the data information in the frequency domain $X(n)$ is partitioned into M clusters as $X(n)$, $(1 \le m \le M)$). All sub-carriers for each cluster are multiplied by the weighting factor, $b_k^m = e^{j\varphi_k^m}$ so as to reduce the PAPR performance. Here, the phase value considered in each cluster is given by the following equation.

$$\varphi_k^m = \frac{2\pi i}{W}, \quad i = 1, 2, W-1 \qquad (6)$$

where W is the number of predetermined discrete phase. After multiplying the weighting factor for each cluster, the subcarrier vector is given by the following equation.

$$x_k = \sum_{m=1}^{M}(b_k^m X_n^m) \qquad (7)$$

where, b_k^m is the weighting factors, which are required to inform the receiver as the side information. The set of weighting factor for M clusters are optimized in the time domain so as to achieve the better PAPR performance, by

$$\hat{x}_k = \sum_{m=1}^{M}(b_k^m \cdot X_k^m) \qquad (8)$$

Finally, at each transmitting antenna, there are (M -1) subblocks to be optimized, and the candidate sequence with the lowest PAPR is individually selected for transmitting. Assume that there are W allowed phase weighting factors. To achieve the optimal weighting factors for each transmitting antenna, W^{M-1} combinations should be checked in order to obtain the minimum PAPR. From (5) to (6), it can be seen that the weighting factor can be multiplied either in the frequency or time domains and the optimized PAPR performance is the same as that given by

$$\tilde{Q} = \arg\min_{0\le w\le W}\max_{0\le n\le L-1}\left|\sum_{m=1}^{M}(b_k^m x_n^M)\right| \qquad (9)$$

where $\tilde{Q}$ is optimized argument that can reduce to the lowest PAPR performance. Then, among all the alternative signals, we choose the signal with the lowest PAPR as the transmitted signal. Therefore the optimal phase rotation sequence $\{\ \hat{b}_0, \hat{b}_1, \cdots, \hat{b}_{M-1}\ \}$ is chosen as

$$\left\{\ \hat{b}_1, \hat{b}_2, \cdots, \hat{b}_{M-1}\right\} = \arg\min_{b_M\in[-1,1]}(\max_{0\le w\le W}\max_{0\le n\le L-1}\left|\tilde{Q}(b)\right|) \qquad (10)$$

As mentioned above, alternate optimization is that all the odd subblocks are kept unchanged and phase weighting factors are optimized only for all the even subblocks, which results in the number of complex multiplications needed for obtaining each candidate sequence being clearly reduced and the number of candidate sequences is also decreased with regard to the same W and M. Therefore, computational complexity is reduced, but PAPR reduction performance is greatly degraded.

3. PAPR REDUCTION SCHEME FOR MIMO-OFDM SYSTEMS

All paragraphs must be indented. All paragraphs must be justified, i.e. both left-justified and right-justified. Nature-inspired methodologies are among the most powerful algorithms for optimization problems. Firefly algorithm (FA) is a novel nature-inspired algorithm inspired by social behavior of fireflies. The FA was developed by Yang [15-16], and is proposed for several different optimization applications. FA is based on this exclusive flashing behaviour of the fireflies. Fireflies are a family of insects which are commonly found in tropical environment. The fireflies are also called lighting bugs, because of their apparent use of bioluminescence during twilight to attract mates or prey. FA has a high speed of convergence. In this paper, a continuous, multi-objective optimization problem has been taken into consideration. Firefly algorithm (FA) [15-17] is metaheuristic algorithms based on the flashing patterns and behavior of fireflies in which the objective function of a given optimization problem is based on differences in light intensity. It helps the fireflies to move towards brighter and more attractive locations in order to obtain optimal solutions. All fireflies are characterized by their light intensity associated with the objective function. Each firefly is changing its position iteratively. The brightness of a firefly is determined by the value of the objective function. For maximization problems, the brightness is proportional to the value of the objective function. Firefly-inspired algorithm uses the following three idealized rules: First, all fireflies are unisex which means that they are attracted to other fireflies regardless of their sex. Secondly, the degree of the attractiveness of a firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. More brightness means less distance between two fireflies. However if any two flashing fireflies are having same brightness, then they move randomly. Finally the brightness of a firefly is determined by the value of the objective function. For a maximization problem, the brightness of each firefly is proportional to the value of the objective function and vise versa. There are two necessary elements, i.e., brightness I and attractiveness β. The I at a particular distance r obeys the inverse square law, which means that I decreases as r increases. In the FA, the brightness of a firefly is expressed in terms of its current position: If it is brighter, its position is preferred; this also means that the value of the objective function is larger. The less bright fireflies will move toward the brighter fireflies. In the case that the brightness of fireflies has the same value, they will move randomly. The brightness of a firefly at some distance r of its location varies with the value of an objective function, which can be defined as

$$I(r) = I_0 e^{-\gamma r^2} \tag{11}$$

where I_0 is the maximum brightness when $r = 0$; it is related to the value of the objective function, and a larger value means brighter. Here, γ is the light absorption coefficient. The attractiveness of a firefly at some distance r of its location is proportional to its light brightness observed by adjacent fireflies; it can be expressed as attractiveness β of the fireflies varies according to the equation given by

$$\beta(r) = \beta_0 e^{-\gamma r^2}, \tag{12}$$

where β_0 is the attractiveness when the distance between two fireflies is zero. In this paper, we simply set $I(r) = \beta(r)$. The distance between two fireflies i and j at positions x_i and x_j can be defined as

$$r_{ij} = \left\| X_i - X_j \right\| = \sqrt{\sum_{k=1}^{l} \left(x_{i,k} - x_{j,k} \right)^2} \tag{13}$$

The equation that updates the jth firefly's location based on the ith firefly's attraction can be described as

$$x_i^j = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_i - x_j) + \alpha(rand - \frac{1}{2}) \tag{14}$$

where x_i and x_j are the initial position of the jth and ith fireflies, respectively, α is a constant within [0, 1], *rand* is a random number within [0, 1], and x_{ij} is the distance between the ith and jth fireflies.

At each iterative step, the brightness and the attractiveness of each firefly is computed. The brightness of each firefly is compared with all other fireflies and the positions of the fireflies are updated using equation (14). In practice the light absorption coefficient γ varies from 0.1 to 1. Owing to the movement of fireflies towards the brighter one, the location and the brightness of the fireflies are updated in the successive iterations of the algorithm. At each iterative step, the brightness and the attractiveness of each firefly is computed. This parameter describes the variation of the attractiveness and its value is responsible for convergence speed of firefly algorithm. In the following, we employ the FA method to search the optimal phase factor for the PTS technique in order to reduce the PAPR.

4. Simulation Results

In this section, some simulations have been conducted to evaluate the capability of the proposed FA-PTS scheme including the PAPR reduction of performance, where 104 data blocks are generated with N = 128. The phase rotation factors are chosen as $b_V \in \{-1,+1\}$. the 16-QAM modulation is employed with *M*=2, 4 and 8, respectively. The side information is assumed to be known at the receiver.

Figure 1 shows that the plot of basic properties of the parameter candidate pool when $C_p = 50$. It was observed that FA has three control parameters, *G* denotes the number of iterations, α is a scale factor and γ the absorption coefficient. The important aspect is to determine the convergence speed of the iterative process. The parameters α and γ, which are related to the convergence speed and the a priori information reliability, respectively, are more influential to algorithm's performance (convergence). All of these parameters affect the speed and robustness of the search with various degrees. The choice of γ which is typically regarded as only a 'fine tuning parameter' is initially recommended to be $0.1 \le \gamma \le 1.0$. In this paper, we suggested that a good initial choice of α is 0.7 or 0.8, *G* = 50 or 100 and γ can be set to 0.8 or 1.0 being a good tradeoff between convergence speeds. Furthermore, linearly increasing and decreasing α during iteration with γ proportional to various combinations of maximum values of the objectives at different iterations.

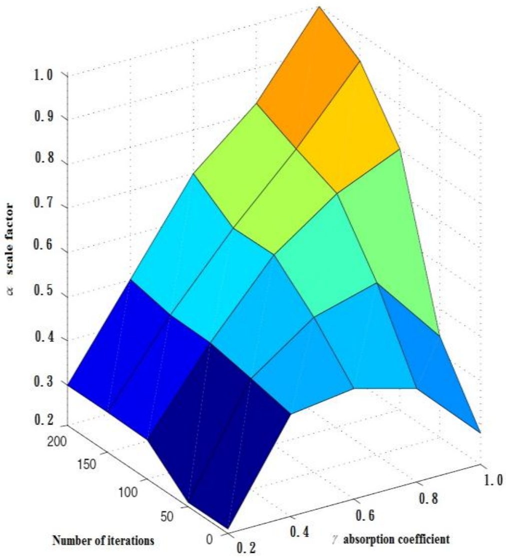

Fig. 1 Plot of basic properties of the parameter candidate pool when $C_p = 50$

The PAPR reduction performance for the MIMO-OFDM is also shown as the purpose of comparison. The total number of sub-carrier (*N*) is 128, number of cluster (*M*) is 2, 4, 8 and 16, and the number of weighting factor (*W*) is 2 phases. In the Figure 3, the PAPR reduction performance is evaluated by using the Complementary Cumulative Distribution Function (CCDF). Figure 2 shows the averaged PAPR performance both for the conventional OFDM method and the proposed method (FA-PTS) when changing the number of iterations. As shown in Figs. 2 and 3, as the maximum number of iterations and γ the absorption coefficient was increased, and the CCDF of the PAPR has been improved. When CCDF of 10^{-3}, we can see that the performance of the proposed FA method provides an approximate PAPR reduction as with that of the conventional ESA.

From this fact, we could confirm that the PAPR performance of MIMO-OFDM signal would be depend on the number of iterations as the same as the conventional OFDM system. The PAPR performance of proposed method can be converged firstly as compared with the CPTS method. Fig. 3, as the maximum number of M is increased, then, the CCDF of the PAPR has a better performance From the Figure 3, it can be seen that the $PAPR_0$ of FA-PTS is 7.5 dB, 8.3dB, 9.3 dB, 10.3 dB and OFDM is 11.6 dB at CCDF of 10^{-4}, respectively. The proposed FA-PTS method shows the better performance.

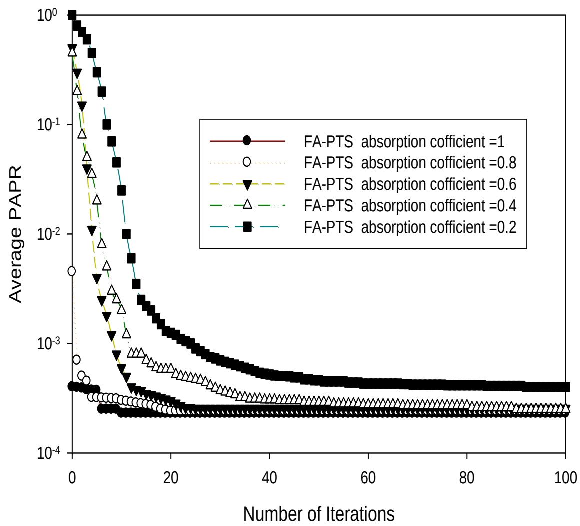

Fig.2 Comparison of average PAPR performance between the proposed method when changing the number of iterations.

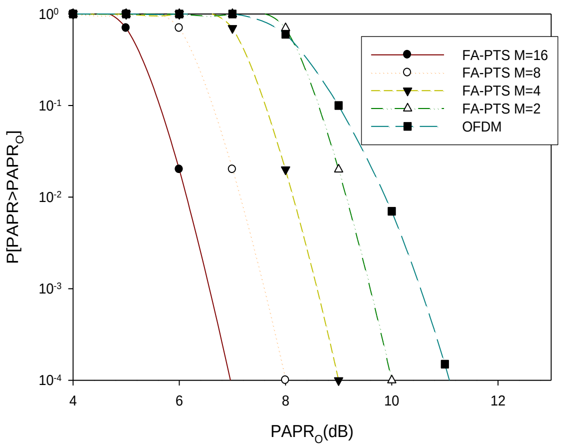

Fig. 3 Comparison of PAPR reduction performance when M=2, 4, 8 and 16.

In Figure 4, some results of the CCDF of the PAPR for FF-PTS technique are shown for the various populations of the fireflies Gn. Here 256 sub-carriers, in which M=8 subblock employing random partition are used with phase weight factor W=2. For population $G_n = 50$, it can be seen that the FA-PTS technique provides improved PAPR performance, with CCDF requirements of 10^{-3}. We can see that the CCDF of the PAPR gradually improves by increasing the population size due to the limited phase weighting factor. As the firefly population size increases, the CCDF of the PAPR improves but the computational complexity also increased at square of the population size. Therefore throughout the simulation, an appropriate population number $G_n = 10$ has been chosen to achieve the best trade-off between the PAPR reduction performance and complexity

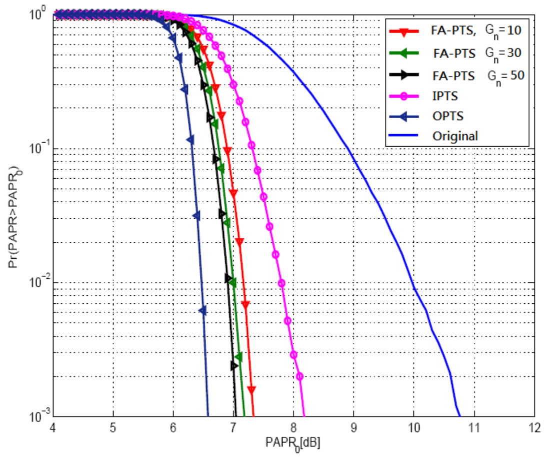

Figure 4. CCDF vs PAPR performance of 16-QAM FF-PTS system for firefly population $G_n = 10$, 30, 50

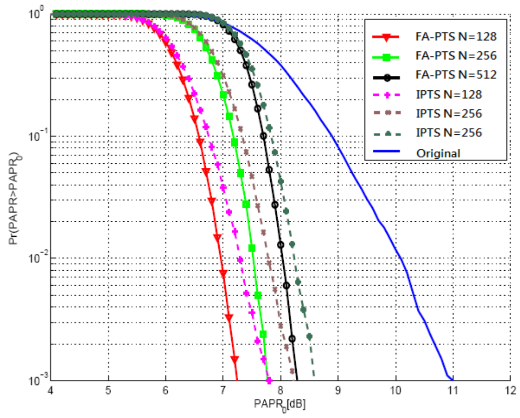

Figure 5. CCDF vs PAPR performance of 16-QAM OFDM-PTS system for N= 128, 256, 512 sub-carriers

Figure 5 presents simulation results of CCDF vs PAPR performance of 16-QAM MIMO-OFDM system for N= 128, 256 and 512 sub-carriers, when M=8 and W=2. Simulation performance for original OFDM, IPTS and FF-PTS algorithm using different number of sub-carriers are presented here. It is seen from the CCDF plot of the PAPR is gradually promoted by increasing the numbers of sub-carriers due to the limited phase weighting factor. As the number of sub-carrier increases, the PAPR also improves. It is also seen, when N= 128, M= 8and W=2, the PAPR of IPTS scheme is approximately 7.8 dB, where as on applying FF-PTS, PAPR reduced to approximately 7.2 dB with CCDF of 10^{-3} . In continuation with this, when N= 256, M= 8 and W=2, after applying IPTS and FA-PTS, PAPR are observed to be 8.2 dB, 7.8 dB respectively with CCDF of 10^{-3}. However with N= 512 sub-carriers, the PAPR of IPTS and FF-PTS are approximately 8.6 dB and 8.3 dB respectively. So, it is observed that quantity improvement in PAPR is dependent on the number of sub-carriers used for OFDM generation, and the optimization algorithms provide improvement in PAPR performance in each case. From the above descriptions, we can see that FA-PTS is a effective technique for reducing the PAPR of MIMO-OFDM system, even with the large number of sub-carriers.

5. CONCLUSION

In this paper, we proposed a novel PTS scheme to reduce the PAPR in Alamouti MIMO-OFDM systems without SI. From the investigations conducted, we conclude that the low-complexity PTS schemes are proposed, which substantially reduce the computational complexity. In order to reduce the PAPR of an MIMO-OFDM signal, a reduced complexity PTS method has been proposed and its derivation is given in this paper considering the relationship between the phase factors and the transmitted signals. Finally, FA based PTS scheme is proposed to search the optimal combination of phase weight factor of OFDM signals.

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