Analytic Interpolator Pilot Pattern Generation Algorithm for Channel Estimation in OFDM Systems

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***Abstract*—In this paper, an algorithm to obtain the near-optimal pilot patterns for analytic interpolator is proposed. Analytic interpolator is superior to the linear and cubic spline interpolation methods on short and nonequidistant pilot patterns, but its performance will be degraded significantly by high noise enhancement factor due to unfitted pilot pattern selection. Therefore, we can improve the performance of analytic interpolation by choosing the pilot patterns that can minify the noise enhancement factor. Under specified pilot density, the near-optimal pilot patterns with least noise enhancement factor can be easily resolved through an algorithm shown in this paper. Simulation results show that the proposed algorithm works near optimally.**

***Keywords*: OFDM, channel estimation, analytic interpolator**

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is the most popular techniques used to support the requirement of high-data-rate transmission in wireless communication systems. For obtaining both a high data rate and high spectral efficiency, however, OFDM systems usually work with multiamplitude modulation schemes such as M-ary QAM that require a more accurate channel estimation to increase the system performance. Generally, from the viewpoint of estimation accuracy, the pilot-aided methods are better than the other methods; however, the pilot-aided methods sacrifice a part of the bandwidth for transmitting pilot symbols [1].

With regard to the pilot-aided methods, the channel estimation needs to not only estimate the channel states on pilot subcarriers but also interpolate the channel frequency response of all the nonpilot channels based on the estimates of the channel state on the pilot subcarriers. Some works about the optimal pilot problem have been published [2-5]. The mean squared error (MSE) of the Wiener channel estimate only relies on the pilot density, not the pilot pattern [6]. However, it may not be practical to use such a Wiener scheme mainly due to the implementation complexity. As a result, a simple interpolator such as a linear Lagrange and spline interpolation scheme is often employed in practice [7-8]. When these interpolators are employed, the performance of the channel estimate is affected by the pilot pattern (i.e., the shape and spacing) as well as the pilot density [8], [9-11]. The optimal pilots to minimize the mean-squared error (MSE) or to maximize channel capacity for single antenna OFDM systems are discussed in [8], [9-10] and those for more general cases such as multiple antenna systems recently in [11]. In summary, minimum mean-squared error (MMSE) optimal pilots should be equal-power and equidistance for single antenna OFDM systems, referred to as comb pilots, while the additional condition of phase-shift orthogonality should be imposed for multiple antenna systems for complete separation of channels associated with different transmitting antennas in the time domain.

The minimum mean squared error optimal pilot pattern can meet the requirement of maximizing the performance of analytic interpolator. However, the optimal pilot pattern presented in [2] can only act under the limitation that the number of subcarriers divided by the number of pilots is integer. This limitation in not practical in effect because that there is a large probability cannot meet this assumption in practice.

In this paper, we release the limitation in [2] and propose a more generalized algorithm to find all optimal pilot patterns on a specified number of subcarriers and pilots no matter the relationship between them.

The rest of this paper is organized as follow. In Section 2, a concise description of analytic interpolator is given. Then, the optimal pilot pattern problem discussion and the optimal pilot patterns generation algorithm are proposed in Section 3. In Section, the proposed algorithm will be verified by numerical simulation, followed by conclusions in Section 5.

2. SYSTEM MODEL

The analytic interpolator and the optimization problem about pilot pattern will be shown briefly. Fig. 1. shows the process for comb-type pilot-aided channel estimation of OFDM systems. The estimate of the channel frequency response on the m-th pilot subcarrier can be given by the least square estimation

$$\hat{\mathbf{H}}_{\mathrm{p}}(m)=\frac{\overline{\mathbf{p}}(m)}{\mathbf{p}(m)}, \qquad (1)$$

where $\overline{\mathbf{p}}(m)$ and $\mathbf{p}(m)$ are the received signal and the prespecified pilot symbol at the m-th pilot subcarrier position. The channel frequency response H is the discrete-time Fourier transform (DFT) of the channel impulse response hL, which is a vector of size L, that is,

$$\mathbf{H}=\mathbf{F}_{NL}\ \mathbf{h}_{L}, \qquad (2)$$

where $\mathbf{h}_L=[h_0,h_1,\cdots,h_{L\text{-}1}]^t$ and FN×L is an N by L matrix comprising the first L column vectors of F, and F is an FFT matrix of size $N\times N$ and is defined as $1/\sqrt{N}[e^{-j2\pi kn/N}$, $0\le k,n\le N-1]$. After some permutation in row on both sides, (2) can be rewritten as

$$\tilde{\mathbf{F}}_{NL}\ \mathbf{h}_{L}=\tilde{\mathbf{H}}. \qquad (3)$$

Here, $\tilde{\mathbf{H}}=\begin{bmatrix}\mathbf{H}_{\mathrm{p}}^t & \mathbf{H}_{\mathrm{d}}^t\end{bmatrix}^t$, where Hp and Hd are vectors of sizes M and N – M, respectively. Apparently, vector Hp contains the elements of the channel frequency response on pilot subcarriers, and Hd contains the elements of the channel frequency response on data subcarriers. Let $\tilde{\mathbf{F}}_{NL}=[\tilde{\mathbf{F}}_{ML}^t\ \tilde{\mathbf{F}}_{ZL}^t]^t$, then the relationship between hL and Hp can be written as

$$\mathbf{h}_{L}=\tilde{\mathbf{F}}_{ML}^{+}\ \mathbf{H}_{\mathrm{p}}, \qquad (4)$$

where $\tilde{\mathbf{F}}_{ML}^{+}=(\tilde{\mathbf{F}}_{ML}^{H}\ \tilde{\mathbf{F}}_{ML})^{-1}\tilde{\mathbf{F}}_{ML}^{H}$ denotes the Moore-Penrose inverse of $\tilde{\mathbf{F}}_{ML}$ [12].

From (1), (2) and (4), we have

$$\hat{\mathbf{H}}=\mathbf{F}_{NL}\ \tilde{\mathbf{F}}_{ML}^{+}\ \hat{\mathbf{H}}_{\mathrm{p}}, \qquad (5)$$

where $\hat{\mathbf{H}}_{\mathrm{p}}=\mathbf{H}_{\mathrm{p}}+\mathbf{n}$.

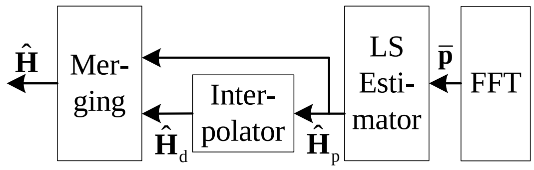

Fig. 1. Process for comb-type pilot-aided channel estimation of OFDM systems.

The error of the analytic interpolator can be given as

$$\mathbf{e}=\hat{\mathbf{H}}-\mathbf{H}=\mathbf{F}_{NL}\ \tilde{\mathbf{F}}_{ML}^{+}\ \mathbf{n}. \qquad (6)$$

Its mean squared error (MSE) is

$$\begin{aligned}\mathrm{MSE}&=\frac{1}{N}E\left[\mathbf{e}^{H}\ \mathbf{e}\right]\\&=\frac{1}{N}tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^{H}\tilde{\mathbf{F}}_{ML}^{+}\right]\sigma_n^2\end{aligned} \qquad (7)$$

where E[u] denotes the expectation of u, tr[Q] refers to the trace of matrix Q and is defined to be the sum of the diagonal elements of Q. In (7), the second equality holds due to the property of iid noise. From (7), it is clear that to minimize the MSE of analytic interpolator is equivalent to minimize the $tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^{H}\tilde{\mathbf{F}}_{ML}^{+}\right]$.

3. PILOT PATTERN GENERATION ALGORITHM

The optimization problem about finding optimal pilot patterns minimizing the MSE shown in (7) for analytic interpolator can be formulated as

$$\begin{aligned}\overline{\mathbf{P}}_{M}&=\left[\overline{\alpha}_i,\ i=0\sim M-1\right]\\&=\arg\underset{\mathbf{P}_M}{\operatorname{minimize}}\ tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^{H}\tilde{\mathbf{F}}_{ML}^{+}\right]\end{aligned}, \qquad (8)$$

in which $\overline{\mathbf{P}}$ denotes the optimal pilot pattern and $\mathbf{P}_M = [\alpha_i, i = 0 \sim M-1]$, $\alpha_i \in \{0,1,...,N-1\}$. In order to develop the optimal pilot patterns generation algorithm, the value of $tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^H \tilde{\mathbf{F}}_{ML}^{+}\right]$ has to be investigated elaborately. After some operation in trace, we obtain

$$tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^H \tilde{\mathbf{F}}_{ML}^{+}\right] = tr\left[(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})^{-1}\right]. \tag{9}$$

The matrix $\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML}$ in (8) can be expanded as

$$\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML} = \frac{1}{N}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ w_N^{-\alpha_0} & w_N^{-\alpha_1} & \cdots & w_N^{-\alpha_{M-1}} \\ \vdots & \vdots & \ddots & \vdots \\ w_N^{-\alpha_0(L-1)} & w_N^{-\alpha_1(L-1)} & \cdots & w_N^{-\alpha_{M-1}(L-1)} \end{bmatrix} \bullet \begin{bmatrix} 1 & w_N^{\alpha_0} & \cdots & w_N^{\alpha_0(L-1)} \\ 1 & w_N^{\alpha_1} & \cdots & w_N^{\alpha_1(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w_N^{\alpha_{M-1}} & \cdots & w_N^{\alpha_{M-1}(L-1)} \end{bmatrix} = \frac{LM}{N}. \tag{10}$$

Assuming that $\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML} = \mathbf{P}\Lambda\mathbf{P}^{-1}$, where Λ is a diagonal matrix of eigenvalues λ_l, $l = 0 \sim L-1$, of $\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML}$ and P is a unitary matrix of the corresponding eigenvectors. Therefore, $tr\left[(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})^{-1}\right] = \sum_{l=0}^{L-1}\frac{1}{\lambda_l}$ and $tr\left[\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML}\right] = \sum_{l=0}^{L-1}\lambda_l = \frac{LM}{N}$. The optimization problem of minimizing $tr\left[\left(\tilde{\mathbf{F}}_{ML}^{+}\right)^H \tilde{\mathbf{F}}_{ML}^{+}\right]$ in (9) is equivalent to maximize $\det(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})$, where $\det(\mathrm{X})$ is the determinant of X.

After some manipulations of mathematics, the optimization problem in (8) can be modified into

$$\bar{\mathbf{D}}_M = \left[\bar{d}_i,\ i = 0 \sim M-1\right] = \arg\underset{\mathbf{D}_M}{\text{maximize}} \prod_{j=0}^{M-1}(d_j)\Big(\sum_{i=j}^{j+1} d_{i \bmod M}\Big)\Big(\sum_{i=j}^{j+2} d_{i \bmod M}\Big).. ..\Big(\sum_{i=j}^{j+(M/2)-2} d_{i \bmod M}\Big)\Big(\sum_{i=k}^{k+\lfloor M/2\rfloor-1} d_{i \bmod M}\Big) \tag{11}$$

in which $\bar{\mathbf{D}}$ denotes the optimal pilot distance pattern and $\mathbf{D}_M = [d_i, i = 0 \sim M-1]$, $d_i \in \{1,...,N-1\}$ and $\sum_{i=0}^{M-1} d_i = N$.

We propose an algorithm for analytic interpolator to generate the pilot patterns minimizing the noise enhancement factor.

The flow chart of proposed algorithm is shown in Fig. 2.

4. Simulation Results and Discussions

In this section, the optimal pilot pattern generation algorithm will be verified by computer simulation. The number of subcarriers N is 16. Firstly, we use exhaustive method to find the optimal pilot patterns with minimum $tr\left[(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})^{-1}\right]$ or maximum $\det(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})$ on M=7 and 10, as shown in Table 1 and 2, respectively. Nest, the optimal pilot pattern list derived form the proposed optimal pilot pattern generation algorithm is shown in Table 3, in which M=7 and 10.

Comparing the optimal pilot patterns list in Table 1 and 2 with Table 3, they are totally identical. Therefore, the proposed optimal pilot pattern generation algorithm can generate all optimal pilot patterns that meet the optimization problem shown in (11).

TABLE 1

Pilot patterns with the least $tr\left[(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})^{-1}\right]$ on M=7.

Pilot Pattern							$tr\left[(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})^{-1}\right]$	$\det(\tilde{\mathbf{F}}_{ML}^H\ \tilde{\mathbf{F}}_{ML})$
1	3	6	8	10	12	15	0.5874	2.2739e+003
0	3	5	7	10	12	14	0.5874	2.2739e+003
1	3	5	8	10	12	15	0.5874	2.2739e+003
0	2	4	7	9	11	14	0.5874	2.2739e+003
0	2	5	7	9	12	14	0.5874	2.2739e+003
1	3	5	7	10	12	14	0.5874	2.2739e+003
0	2	4	6	9	11	13	0.5874	2.2739e+003
1	3	5	8	10	12	14	0.5874	2.2739e+003
1	4	6	8	11	13	15	0.5874	2.2739e+003
0	2	4	7	9	11	13	0.5874	2.2739e+003
0	2	5	7	9	11	14	0.5874	2.2739e+003
1	3	6	8	10	13	15	0.5874	2.2739e+003
0	3	5	7	9	12	14	0.5874	2.2739e+003
1	4	6	8	10	13	15	0.5874	2.2739e+003
2	4	6	8	11	13	15	0.5874	2.2739e+003
2	4	6	9	11	13	15	0.5874	2.2739e+003
1	3	5	7	9	12	15	0.5881	2.2705e+003
1	3	6	9	11	13	15	0.5881	2.2705e+003

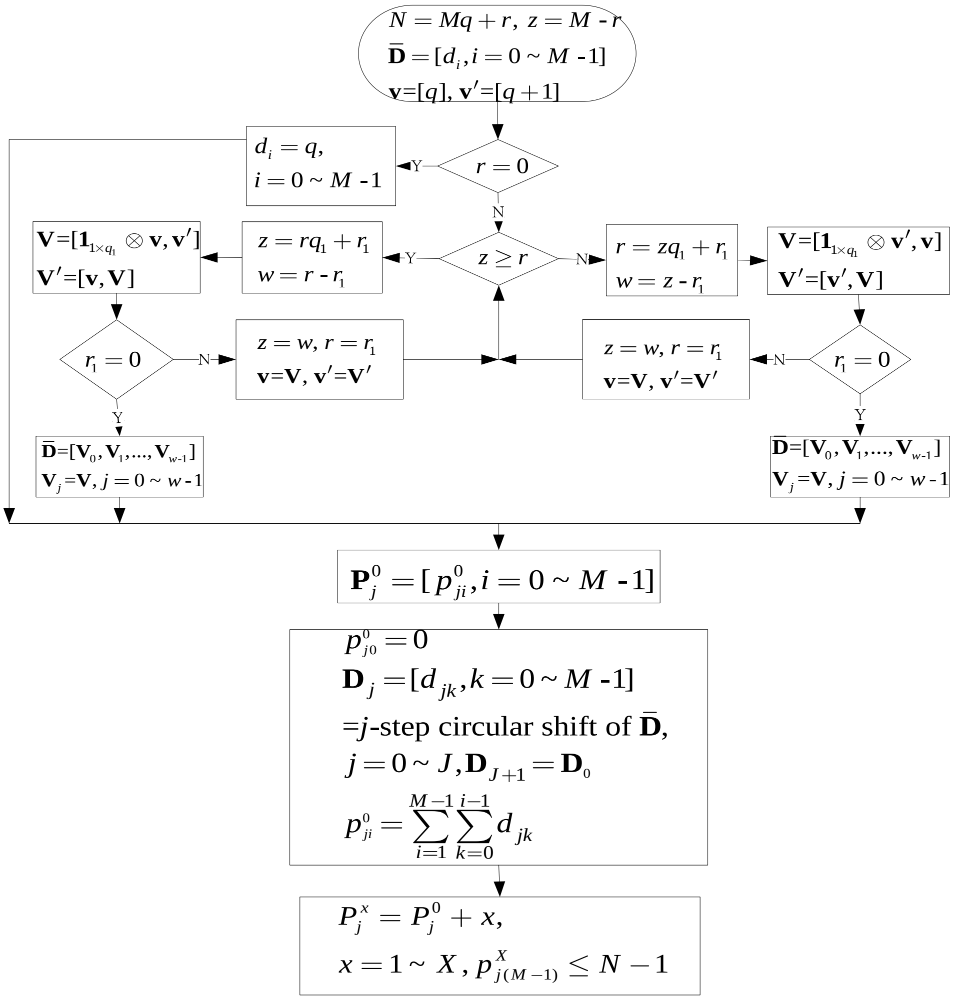

Fig. 2. Optimal pilot pattern generation algorithm.

TABLE 2

Pilot patterns with the least $tr\left[(\tilde{\mathbf{F}}_{ML}^H \tilde{\mathbf{F}}_{ML})^{-1}\right]$ on M=10.

Pilot Pattern										$tr\left[(\tilde{\mathbf{F}}_{ML}^H \tilde{\mathbf{F}}_{ML})^{-1}\right]$	$\det(\tilde{\mathbf{F}}_{ML}^H \tilde{\mathbf{F}}_{ML})$
0	**1**	**3**	**4**	**6**	**8**	**9**	**11**	**12**	**14**	**0.4028**	**9.8632e+003**
0	**1**	**3**	**5**	**6**	**8**	**9**	**11**	**13**	**14**	**0.4028**	**9.8632e+003**
0	**2**	**3**	**5**	**6**	**8**	**10**	**11**	**13**	**14**	**0.4028**	**9.8632e+003**
0	**2**	**3**	**5**	**7**	**8**	**10**	**11**	**13**	**15**	**0.4028**	**9.8632e+003**
0	**2**	**4**	**5**	**7**	**8**	**10**	**12**	**13**	**15**	**0.4028**	**9.8632e+003**
1	**2**	**4**	**5**	**7**	**9**	**10**	**12**	**13**	**15**	**0.4028**	**9.8632e+003**
1	**2**	**4**	**6**	**7**	**9**	**10**	**12**	**14**	**15**	**0.4028**	**9.8632e+003**
1	**3**	**4**	**6**	**7**	**9**	**11**	**12**	**14**	**15**	**0.4028**	**9.8632e+003**
0	1	3	4	6	7	9	11	12	14	0.4040	9.8076e+003
0	1	3	4	6	8	9	11	13	14	0.4040	9.8076e+003
0	1	3	5	6	8	9	11	12	14	0.4040	9.8076e+003
0	1	3	5	6	8	10	11	13	14	0.4040	9.8076e+003

TABLE 3

Optimal pilot pattern list with N=16, M=7 and 10.

M	7						
$\bar{\mathbf{D}}$	[2	2	3	2	2	2	3]
P	0	2	4	7	9	11	13
	0	2	5	7	9	11	14
	0	3	5	7	9	12	14
	0	2	4	6	9	11	13
	0	2	4	7	9	11	14
	0	2	5	7	9	12	14
	0	3	5	7	10	12	14
	1	3	5	8	10	12	14
	1	3	6	8	10	12	15
	1	4	6	8	10	13	15
	1	3	5	7	10	12	14
	1	3	5	8	10	12	15
	1	3	6	8	10	13	15
	1	4	6	8	11	13	15
	2	4	6	9	11	13	15
	2	4	6	8	11	13	15

M	10
$\bar{\mathbf{D}}$	[2 1 2 2 1 2 1 2 2 1]
P	0 2 3 5 7 8 10 11 13 15 0 1 3 5 6 8 9 11 13 14 0 2 4 5 7 8 10 12 13 15 0 2 3 5 6 8 10 11 13 14 0 1 3 4 6 8 9 11 12 14 1 2 4 6 7 9 10 12 14 15 1 3 4 6 7 9 11 12 14 15 1 2 4 5 7 9 10 12 13 15

5. CONCLUSIONS

The optimal pilot pattern for analytic interpolator can mitigate the noise enhancement significantly. This pattern is not unique, and the key point to select the optimal pattern is the distance between adjacent pilots. To find optimal pilot pattern is an optimization problem on minimizing trace of a specific matrix. Through a series of manipulation of matrix, the optimization problem has been reduced to a simpler equivalent problem of pilot distance permutation. The algorithm generating all near-optimal pilot patterns derived from the reduced optimization problem has illustrated. The proposed algorithm has verified through numerical simulation.

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