

EXIT-Chart Based Labeling Design for Type-I HARQ in BICM-ID Systems

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Abstract— This paper investigates the issue of labeling design in the BICM-ID (bit-interleaved coded modulation-iterative decoding) systems with Type-I HARQ (hybrid auto-retransmission request). Based on the EXIT (extrinsic information transfer) chart, an algorithm is developed in search of good labelings that well match with the outer code. The concept of link adaptation is realized by using different labelings at different SINR (signal-to-noise ratio) to achieve a better performance. Numerical results show that the new design outperforms the existing ones.

Keywords— BICM-ID, HARQ, Labelling Design, EXIT Chart

1. INTRODUCTION

BICM (bit-interleaved coded modulation) has been known as an effective coded modulation scheme for the fading environment. By introducing a bit inter-leaver between encoder and modulator, the code diversity can be increased, and thus improving the link performance [1]-[3]. BICM has been employed in real wireless systems such as the HSDPA [4] and WiMAX [5] systems.

Hybrid ARQ (auto-retransmission request) is an important technique in today's wireless systems to improve link performance [6],[7]. It combines the forward error control coding (FEC) and ARQ. Roughly speaking, there are three types of HARQ schemes, including Chase combining (Type-I) [6], full incremental redundancy (Type-II) [7] and partial incremental redundancy (Type-III). HARQ provides high link reliability and thus can improve the system throughput very substantially. Both the HSDPA

and WiMAX systems employ this technique to obtain good performance.

EXIT chart proposed by Brink [8] is a powerful tool to analyse convergence behaviour of iterative decoding. It uses mutual information between extrinsic output and the corresponding bits to predict the performance of iterative decoding system precisely. Frequently, EXIT chart is used to design turbo code, LDPC code and BICM-ID systems.

In this paper, EXIT chart is employed to design Type-I HARQ in the BICM-ID systems. In particular, new labelings that well match the outer code are designed to improve performance. Using the labelling in [9] for the first transmission, an algorithm is proposed to search for good labelings for the subsequent transmissions. Link adaptation is realized by using different labelings at different SNRs to further improve the performance.

2. SYSTEM MODELS

2.1. Type-I HARQ

In Type-I HARQ, if receiver fails to decode the transmitted packet, a retransmission request, NACK, is fed back from receiver to transmitter. Upon reception of the NACK, the transmitter sends the same packet again. The receiver needs to buffer all previously received packets and combine those with the recently received packet according to maximal ratio combining (MRC), which was first discussed by Chase in [6]. Hence, Type-I HARQ is also referred to Chase combining (CC).

Type-I HARQ provides diversity gain by decoding a packet with multiple received signals. To perform Type-I HARQ, it needs only

additional buffer to save received packets, as compared with the conventional ARQ, but the feedback and decoding schemes are the same as those in the conventional ARQ.

2.1.1. Throughput of Type-I HARQ

A key performance metric for HARQ is its throughput, which is the number of bits conveyed per unit time. In Type-I HARQ, the information we want to deliver are encoded as binary coded bits sequence, namely, data packet. Suppose the modulation scheme is identical during retransmission, the throughput of Type-I HARQ can then be calculated as

$$\Psi = \frac{N_s}{N_T}, \quad 0 \leq \Psi \leq 1, \quad (1)$$

where N_T is the total number of data packets sent, and N_s is the number of packet received successfully.

2.2 HARQ in BICM-ID

Fig. 1 shows the system model of the BICM systems with Type-I HARQ. A binary information sequence $\mathbf{b}$ is encoded as a coded sequence $\mathbf{c}$, and a bit level inter-leaver Π located between the encoder and modulator is to convert $\mathbf{c}$ to a bit sequence $\mathbf{c}'$. The modulation scheme has an M -ary constellation signal set $\mathcal{X} = \{x_1, \dots, x_M\}$ with $M = 2^m$. $\mathbf{c}'$ is put into the modulator sequentially and every m bits are grouped to form a label $\mathbf{s} = [c_0, \dots, c_{m-1}]$. Let Λ be the set of all possible labels. Through the modulator, a constellation point $x = \mu(\mathbf{s})$, $x \in \Lambda$ is selected according to a labeling mapping function μ . μ_i represents the mapping function used in the i -th transmission.

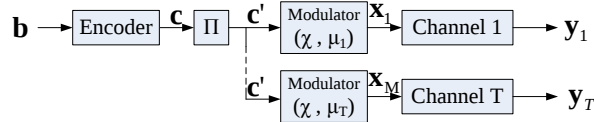


Fig. 1 BICM system with Type-I HARQ

2.2.1 Bit Interleaver

The inter-leaver used in the system is the S-random inter-leaver [10]. An S-random inter-leaver guarantees that the two bits within a

distance S_1 at the inter-leaver input cannot be mapped to a distance less than S_2 apart at the inter-leaver output. Usually $S_1 = S_2 = S$.

2.2.2 Modulator

The coded bits coming out the bit inter-leaver are grouped every m bits and mapped to constellation point by specific mapping function. In the paper, we consider only 16-QAM and 8PSK constellations as shown Fig. 2.

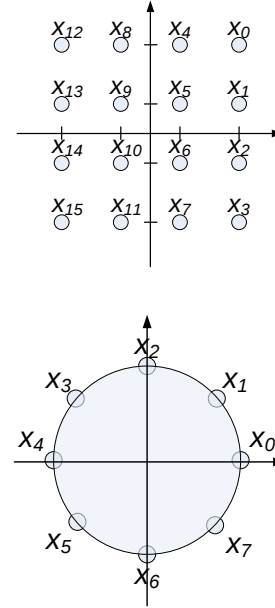


Fig. 2 16QAM and 8-PSK constellations

2.2.3 Iterative Decoding

Since the optimal MAP decoding for the BICM system is too complex to implement, we separate the receiver into two parts : detector and decoder, and the detector and the decoder exchange soft information iteratively.

Fig. 3 shows a diagram of iterative decoding for the BICM Type-I HARQ systems. The detector ϕ takes channel outputs $\mathbf{y}_1, \dots, \mathbf{y}_M$, and intrinsic LLRs from decoder, $\mathbf{L}_A^{(\phi)}$, to compute extrinsic LLRs, $\mathbf{L}_E^{(\phi)}$. Then, the extrinsic LLRs of detector are de-interleaved and serve as a priori inputs $\mathbf{L}_A^{(\phi)}$ of decoder.

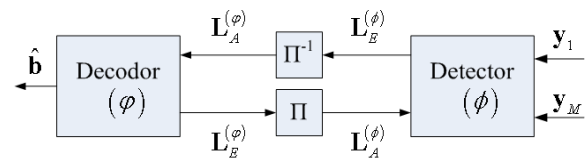


Fig. 3 BICM-ID

2.2.4 Detector and Decoder

The MAP detector is used to compute the a posteriori probability for coded bits. Since memory-less modulation scheme is used, we demodulate received signal symbol by symbol. The likelihood ratio for bit c_i in a symbol,

$$\begin{aligned} L_D^{(\phi)}(i) &= \log \frac{\Pr(c_i = 1 | y_1, \dots, y_T)}{\Pr(c_i = 0 | y_1, \dots, y_T)} \\ &= \log \frac{\Pr(c_i = 1)}{\Pr(c_i = 0)} + \log \frac{\Pr(y_1, \dots, y_T | c_i = 1)}{\Pr(y_1, \dots, y_T | c_i = 0)}, \quad (2) \\ &= L_A^{(\phi)}(i) + L_E^{(\phi)}(i) \end{aligned}$$

where

$$L_E^{(\phi)}(i) = \log \frac{\sum_{s \in \Lambda_i^1} \exp \left(\sum_{j=0}^{m-1} c_j^s L_A^{(\phi)}(j) \right) \exp \left(\frac{\sum_{k=1}^T \|y_k - \hat{h}_k \mu_k(s)\|^2}{\sigma^2} \right)}{\sum_{s \in \Lambda_i^0} \exp \left(\sum_{j=0}^{m-1} c_j^s L_A^{(\phi)}(j) \right) \exp \left(\frac{\sum_{k=1}^T \|y_k - \hat{h}_k \mu_k(s)\|^2}{\sigma^2} \right)}, \quad (3)$$

$\hat{h}_k$ is the channel gain for the i -th transmission, Λ_i^b is the set of labels with i -th bit equal to b , and c_j^s is j -th bit of label s .

In the BICM-ID system, a SISO (soft-in, soft out) decoder is needed to compute soft information passed to detector. In the paper, BCJR algorithm [11] is utilized.

2.2.5 Exit Chart

EXIT chart was proposed by S. ten Brink in 2001. EXIT chart depicts the relation between mutual information of intrinsic log-likelihood ratios and coded bits and that of extrinsic log-likelihood ratios and coded bits for decode (or detector). Using mutual information transfer characteristics of SISO decoder, we can analyse the convergence behaviour and design the system for better performance.

3. Search Algorithm

In this section, we propose an algorithm to search for labeling for Type-I HARQ in order to make throughput as high as possible. First, an analytical model for EXIT chart is introduced. With the simple model, the detector transfer curve on EXIT could be approximated accurately

by a close-form function. Next, we propose a search algorithm using the concept of binary search algorithm to establish a labeling set from which the best labeling can be chosen according to code scheme and SNR.

3.1. Simplified Model

An analytical approximate expression for the EXIT function of a maximum a posteriori probability detector over memory-less channels under single transmission is derived in [13]. The real communication channel is replaced with a hard decision virtual channel, and the intrinsic information coming from decoder is modelled as a binary symmetric channel or a binary erasure channel. The method was modified to the case of HARQ in [14].

3.2. Search Algorithm

Consider a HARQ system, the mapping function for the first transmission can be one of the mapping function obtained in previous work [9] which is designed to match the outer code. Suppose the mapping functions of $T-1$ transmissions have been known, we want to design the mapping function for T transmission, $\mu^T(s)$, based on mapping functions $\mu^1(s), \dots, \mu^{T-1}(s)$.

Define $I_0^\gamma(\mu_1, \dots, \mu_T)$ as the mutual information between detector extrinsic and codeword under simplified model when the mutual information between detector intrinsic and codeword is equal to zero, that is, it relates to the leftmost point on EXIT chart, and $I_1^\gamma(\mu_1, \dots, \mu_T)$ is the mutual information between detector extrinsic and codeword under simplified model when the mutual information between detector intrinsic and codeword is equal to one, that is, it relates to the rightmost point on EXIT chart. From observation, the detector transfer curve approaches a straight line. Therefore, a straight line from $(0, I_0^\gamma(\mu_1, \dots, \mu_T))$ to $(1, I_1^\gamma(\mu_1, \dots, \mu_T))$ is defined as a virtual transfer curve and the slope of the virtual transfer curve is defined as “slope” of the jointly-detected transfer curve of the mapping function pair $\mu_1, \dots, \mu_T$. Another reason to introduce the virtual transfer curve is that $(0, I_0^\gamma(\mu_1, \dots, \mu_T))$ is the point that decides whether a tunnel exists between detector

transfer curve and decoder curve such as the trajectory can stretch far away, and $(1, I_1^\gamma(\mu_1, \dots, \mu_T))$ is the point decides the position of error flow. Hence, the two points on EXIT chart is what we are mostly concerned.

First, base on predetermined mapping functions $\mu^1(s), \dots, \mu^{T-1}(s)$, we set up a table as TABLE I. Because the slope of detector curve is from 0 to 1, we quantize those uniformly to finite set $\mathbf{M}$, $|\mathbf{M}| = K$, that is, in row k , m_k is equal to $\frac{k}{K} \cdot \mu_k$ in row k is the mapping function such as

$$m_k = \arg \min_{m \in \mathbf{M}} \left\{ \left[\begin{array}{c} I_0^\gamma(\mu^1, \dots, \mu^{T-1}, \mu_k) \\ -I_0^\gamma(\mu^1, \dots, \mu^{T-1}, \mu_k) \end{array} \right] - m \right\},$$

$$D_{\mu_k} = I_0^\gamma(\mu^1, \dots, \mu^{T-1}, \mu_k) + I_0^\gamma(\mu^1, \dots, \mu^{T-1}, \mu_k)$$

namely, m_k is the quantized slope of virtual transfer curve of $(\mu^1, \dots, \mu^{T-1}, \mu_k)$, and D_{μ_k} stands for the area under the virtual transfer curve because the altitude of the ladder-shaped area is equal to one.

TABLE 1

TABLE USED IN SEARCHING ALGORITHM

	M	D	
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	m_k	D_{μ_k}	μ_k
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The algorithm goes like the following.

Initial Step: Predetermine a number of initial seeds. A table is set up for each seed with every D_{μ_k} value set to zero initially. An initial seed mapping function is given in this step sequentially.

Step. 1: Compute all possible swaps of two indices for all seeds, and update the table belong to the initial seed. For instance, to deal with 3 seeds with 16QAM modulation, there are $3 \cdot (16 \cdot 15 / 2) = 260$ possible swaps. The method to update the table listed as follow :

Suppose Σ is the set composed of all swaps, than compute all $\mu \in \Sigma$

$$D = I_1(\mu^1, \dots, \mu^{T-1}, \mu) + I_0(\mu^1, \dots, \mu^{T-1}, \mu)$$

$$m' = I_1(\mu^1, \dots, \mu^{T-1}, \mu) - I_0(\mu^1, \dots, \mu^{T-1}, \mu)$$

$$m_k = \arg \min_{m \in \mathbf{M}} (\|m' - m\|)$$

If $(D > D_{\mu_k})$

Update the table $D_{\mu_k} = D$ and $\mu_k = \mu$

After all swaps have been calculated, go to step.2.

Step. 2:

If (the table has been updated)

Set the labels correspond to the updated rows as new seeds and return to Step. 1.

if (the table has not been updated)

{The table related to this initial is kept.

if (new initial seed is available)

Pick a new initial seed with corresponding table and return to Step. 1.

else

Go to Step. 3}

Step. 3: Compare all the tables related to each initial seed we obtained above. We pick up the mapping functions with maximum D_{μ_k} value among each specific row k of all the tables. Those picked-up mapping functions are formed a mapping function set of which detector transfer with various slopes and largest area.

Perform searching algorithm depicted above, a mapping functions set for retransmission can be obtained sequentially. With the mapping functions set, link adaptation can be realized by that using distinct mapping function according to channel signal to noise ratio (SNR).

4. SIMULATION RESULTS

In this section, a set of mapping functions for retransmissions are obtained through the proposed searching algorithm. The performance of our design is compared with that of chase combining and previous labeling that is obtained with no consideration of the outer code. The parameters of simulation are listed in TABLE II.

4.1 CC (133₈, 171₈) , 16QAM, Rayleigh Channel

The optimal mapping function of first transmission for (133₈ , 171₈) convolution code is picked from the labeling set in [9]. Base on the mapping function we search the mapping function for the second transmission using the

proposed searching algorithm. The results are shown in Fig. 4, where two mapping functions are selected. It is observed that the two mapping function have different properties, for example, the left point of the “red” one is higher than the “pink” one, and we suppose the performance of “red” one is with better than that of the “pink” one at low SNR region. On the other hand, the “pink” one is better than the “red” one at high SNR region. The transfer functions for third transmission are based on the first mapping function and the second mapping function with “red” transfer curve.

Fig. 5 shows the throughput of our design, chase combining and the labeling obtained in [15]. In [15], the authors searched retransmission mapping function based on the BER upper bound under zero-prior and ideal-prior conditions and without considering outer code. We compare three HARQ strategies using identical mapping function of first transmission. In Fig. 3, the chase combining performs the worst because it only utilizes power gain with no diversity gain. The results from method in [15] and our algorithm both utilize the power gain and symbol diversity gain, and the result of our algorithm utilize the diversity gain more accurately because of using the powerful tool EXIT chart.

TABLE II
SIMULATION PARAMETER OF CC (133₈, 171₈)

Coding scheme	(133 ₈ , 171 ₈) conculational code
code rate	1/2
Modulation scheme	16QAM, 8PSK
# of information bits	2500
BICM - ID Iteration (Outer Iteration)	10

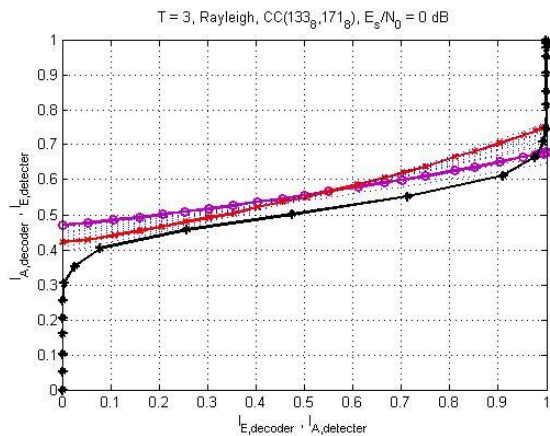


Fig. 4 Transfer curves of the suitable labeling in our design for CC (133₈, 171₈) with 16-QAM.

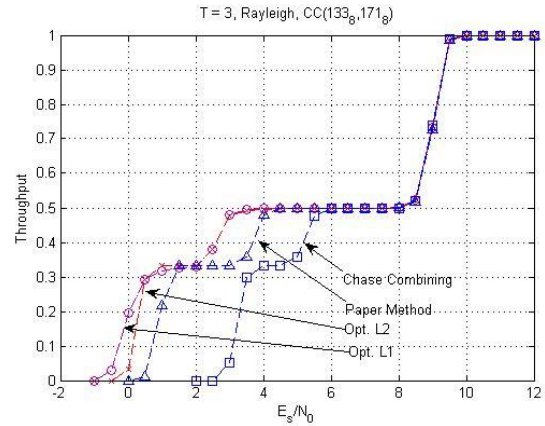


Fig. 5 Throughput of the suitable labeling for CC (133₈, 171₈) with 16-QAM under T = 3

4.2 CC (133₈, 171₈), 8PSK, Rayleigh Channel

In this part, the results of 8-PSK for CC (133₈, 171₈) under Rayleigh fading channel are given. Figs. 6 and 7 shows the throughput with T = 2 and T = 3, respectively. It is clear that the performance of our design has about 2 dB gain over the chase combining.

5. CONCLUSIONS

In this paper, we developed a systematic algorithm to search for good labelings for retransmissions in the Type-I HARQ systems for the BICM-ID systems. Link adaptation with different labeling for retransmission is proposed with 2-3 dB performance improvement over the traditional chase combining scheme..

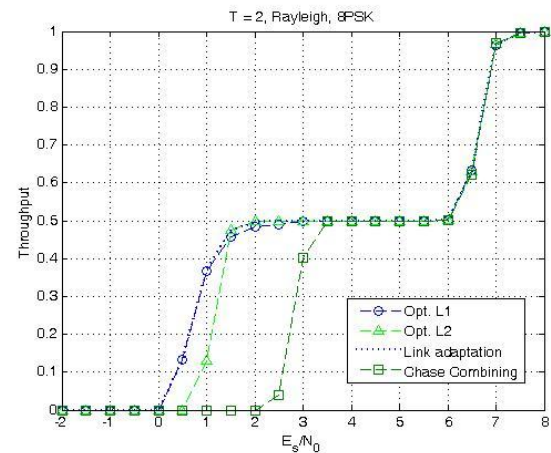


Fig. 6 Throughput of the selected labeling for CC (133₈, 171₈) with 8-PSK under T = 2

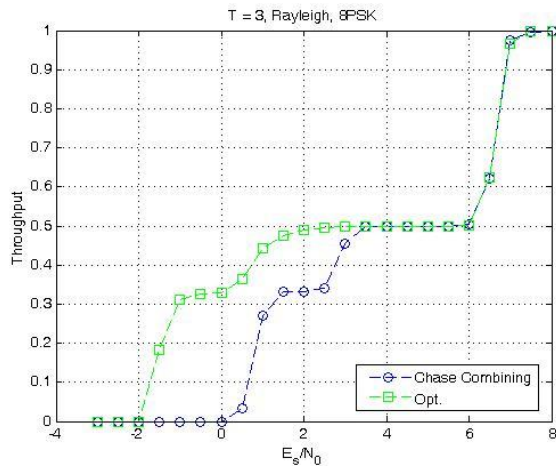


Fig. 7 Throughput for the selected labeling for CC (133₈, 171₈) with 8-PSK with 8-PSK under $T = 3$

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