Combine Block Coded Modulation Codes with M-PTS for Peak-to-Average Power Ratio Reduction in OFDM Systems

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Abstract— **This paper proposes a novel peak-to-average power ratio (PAPR) reduction method with error correction capability in a 16-QAM orthogonal frequency division multiplexing (OFDM) system, called BCM-MSM-PTS. The M-PTS scheme, which is combined the multi-points square mapping (MSM) scheme with the conventional partial transmit sequence scheme, has been proposed to improve the PAPR performance in the OFDM systems. Although the M-PTS scheme has many advantages, the transmitted signal does not have error correction capability. Therefore, this paper proposes a modified M-PTS with error correction capability, called BCM-MSM-PTS, that combines block coded modulation (BCM) codes with the M-PTS technique. Note that each QPSK BCM code is composed of two Reed-Muller codes. Simulation results provide that the best selection of QPSK BCM codes to improve the PAPR performance of 16-QAM OFDM systems.**

Keywords— **OFDM, BCM, PAPR, M-PTS, Reed-Muller codes.**

1. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) technique, which is an efficient transmission technique, has been adopted in several wireless standards [1]. One major drawbacks of the OFDM technique is high peak-to-average power ratio (PAPR) of the OFDM signals. Due to the high PAPR decreases the efficiency of the high power amplifier (HPA) and increases the complexity of the analog-to-digital (A/D) converter, this results has motivated the seek for the PAPR reduction methods.

Recently, many solutions have been proposed to improve the high PAPR in OFDM systems, such as block coding [2], selective mapping (SLM) [3][4], partial transmit sequence (PTS) [5]-[7], and active constellation extension (ACE) [8][9] etc.

The PTS scheme is an efficient method to improve the problem of high PAPR. First, the PTS scheme divides the modulated signals into several disjoint sub-blocks. Next, multiply these sub-blocks with the phase rotation factors to generate the candidates. The OFDM signal with minimum PAPR is selected from these candidates.

However, the transmitter of the PTS scheme needs to transmit the additional message, called side information, to recover the information bits in the receiver. Therefore, eliminate need to transmit the side information has become an active research in the PTS scheme. Recently, the multi-points square mapping combined with PTS (M-PTS) [10] technique has been proposed in 16-QAM OFDM systems and does not need to transmit the side information. However, the PTS and M-PTS scheme do not have error correction capability against channel effects. Therefore, this paper proposes a modified M-PTS scheme, called BCM-MSM-PTS, to improve the error correction capability of the M-PTS scheme. Also, the BCM-MSM-PTS scheme does not need to transmit the side information.

2. PRELIMINARIES

2.1. PAPR in OFDM

For an OFDM system each modulation symbol is mapped into a single subcarrier. The signal is produced by a block of N data symbols $\mathbf{X} = \{X_n, n = 0,1,...,N-1\}$. The mathematical formula of an OFDM signal can be described as

$$x(t)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}X_n e^{j2\pi\frac{n}{T}t}, 0\le t<T. \quad (1)$$

where *1/T* is the bandwidth of each subcarrier.

The PAPR of the transmitted signal x(t) is defined as the ratio of the maximum to the average power during an OFDM signal duration, expressed as

$$PAPR\{x(t)\}=\frac{\max_{0\le t\le T}[|x(t)|^2]}{E[|x(t)|^2]}, \quad (2)$$

where $E[|x(t)|^2]$ is the average power of x(t).

In general, to approximate the continuous PAPR of x(t), the samples of OFDM signal are obtained by its discrete LN samples $x(iT/LN)$, i=0,1,…,LN-1. That is, these samples are obtained from the LN-point IFFT of $\{X_0,X_1,\ldots,X_{N-1}\}$ with $(L-1)N$ zero-padding. This paper uses L=4 for numerical simulation, which is enough to estimate the continuous PAPR. In addition, the complementary cumulative distribution function (CCDF) of PAPR is always used for measuring the performance of PAPR reduction techniques. Given a reference PAPR level $PAPR_0$, the CCDF of PAPR is the probability that the PAPR of transmitted signal exceeds $PAPR_0$. In this paper, the CCDF of PAPR is written as CCDF(PAPR) = $Pr(PAPR>PAPR_0)$.

2.2. M-PTS

In order to explain the principle of the MSM scheme [10], Fig. 1 is a good example. There is a point at (1+i), after multiplied by [1,-1,i,-i], the new points are (1+i), (-1+i), (-1-i), (1-i), respectively. Obviously, the four new generated points in the constellation represent the original point extension.

In the conventional PTS technique over QPSK modulation, the input block $\boldsymbol{X}$ of length N is partitioned into M disjoint sub-blocks $\boldsymbol{X_i}$ of length N, i=1,2,…,M, which can be represented as $\boldsymbol{X}=\sum_{i=1}^{M}X_i$. Therefore, the disjoint sub-blocks are transformed from the frequency domain to time domain after IFFT and multiplied by the phase rotation factor. That is, these OFDM signals can be written as

$$x_b(t)=\frac{1}{\sqrt{N}}\sum_{i=0}^{M}B_i X_i e^{j2\pi\frac{n}{T}t}, \quad (3)$$

where $B_i = e^{j2\pi\omega/W}$, ω=0,1,…,W-1, and W is the number of allowed phase angles. It is possible to generate W^M different OFDM signals and selecting the minimum PAPR by the optimal phase factor to transmit signals.

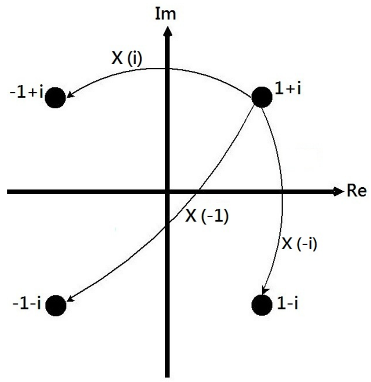

Fig. 1 Different representing of one point with different number.

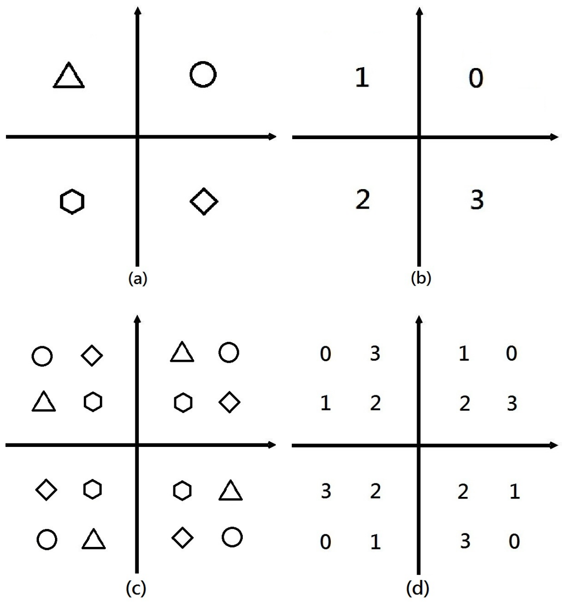

Fig. 2 (a) the original 4-QAM modulation points; (b) the 4 points of information expressed; (c)the generated 16 points in constellation; (d) the 16 points of information expressed.

The M-PTS [10] is combining the MSM scheme with the conventional PTS technique. In the traditional PTS, supposedly the phase rotation factor is [1,j,-1,-j], and by using 4-QAM modulation for mapping data into a single subcarrier. The original 4-QAM modulation points (shown in Fig. 2(a)) would generate a new 16-QAM points (shown in Fig. 2(d)) in a constellation via linear combination through the phase rotation factor [1,j,-1,-j] by MSM scheme. However, the 16-QAM points represent an extension of the original 4-QAM points. Then, it selected the signals with the minimum PAPR to transmit signals. Fig. 2(b) and Fig. 2(d) show the 4-QAM and 16-QAM modulation points of information expressed.

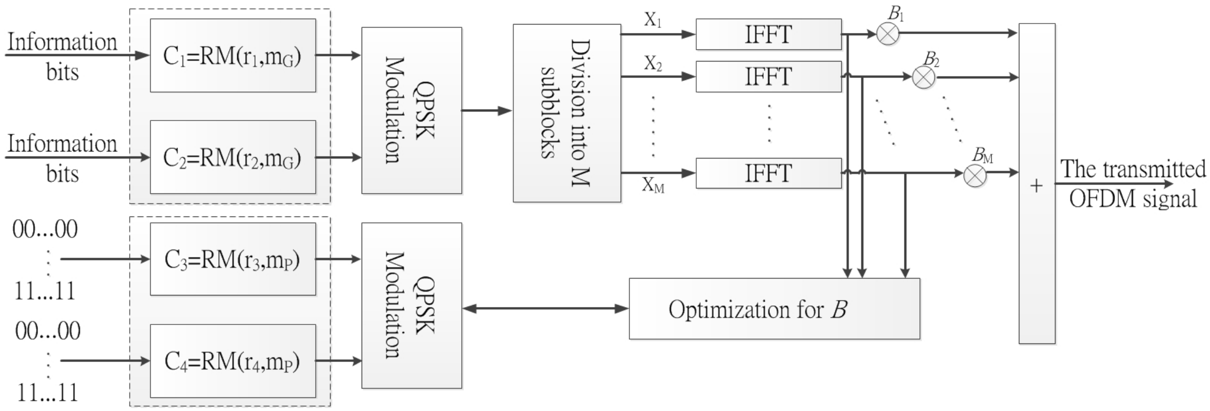

Fig. 3 The block diagram of a BCM-MSM-PTS OFDM system.

2.3. Block Coded Modulation

Let C_1, C_2, C_3 and C_4 be four binary codes of length n with minimum Hamming distance d_i, $1 \le i \le 4$, respectively. A $4 \times N$ codeword array is constructed by filling the ith row with codeword from the ith component code C_i as follow:

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \in C_1 \\ \mathbf{c}_2 \in C_2 \\ \mathbf{c}_3 \in C_3 \\ \mathbf{c}_4 \in C_4 \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,N} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,N} \\ c_{3,1} & c_{3,2} & \cdots & c_{3,N} \\ c_{4,1} & c_{4,2} & \cdots & c_{4,N} \end{bmatrix}.$$

Then, a complex-valued sequence of length N in a 16-QAM BCM code B can be formed by modulating each column of the $4 \times N$ codeword array into a 16-QAM symbol. The 16-QAM signal constellation with bit labels is assigned by a set partitioning scheme in which the intra-subset squared Euclidean distances rise in the ratio 1:2:4:8.

More precisely, a 16-QAM BCM codeword b = $[b_0\ b_1\ \ldots\ b_{N-1}] \in B$ can be written as

$$b_k = \left(\frac{1}{\sqrt{2}} j^{c_{1,k}+2c_{2,k}} + \sqrt{2} j^{c_{3,k}+2c_{4,k}}\right) e^{j\pi/4}, 0 \le k \le N-1,$$

in terms of these four binary component codeword $c_i \in C_i$. The minimum squared Euclidean distance between any two codewords in B is known as $D_E^2 = \min_{1 \le i \le 4} d_i \cdot 2^{i-1} \cdot E^2$, where E is the minimum Euclidean distance between signal points [11]. To increase the codes rate and balance the minimum squared Euclidean distance of a 16-QAM BCM code, we require d_1=2d_2=4d_3=8d_4. For example, we can take the component code C_i=RM(i,m), $1 \le i \le 4$.

3. BCM-MSM-PTS SCHEME

The process of BCM-MSM-PTS is first to divide four Reed-Muller codes into two groups for generating two BCM QPSK modulation. We use two binary Reed-Muller codes to construct the BCM codes for a QPSK modulation and use two BCM QPSK modulation in the 16-QAM constellation. One QPSK modulation uses to information data coding from the BCM codes, and the other one uses to produce phase rotation factor. Then, the BCM-MSM-PTS disjoints the component codes C_1, C_2, C_3 and C_4. The C_1 and C_2 are using to construct the BCM codes for the BCM QPSK modulation and considering the PAPR performance for Reed-Muller codes in random ordering**,** natural ordering and cyclic ordering. The C_3 and C_4 are constructing the BCM codes for the other BCM QPSK modulation and generating the phase rotation factor. Therefore, two generator matrix **G** and **P** can be written as

$$\mathbf{G} = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}, \ \mathbf{P} = \begin{pmatrix} C_3 & 0 \\ 0 & C_4 \end{pmatrix},$$ where **G** can be a BCM QPSK modulation from information data; and **P** can be the phase rotation factor as [1,j,-1,-j].

Fig. 3 shows the block diagram of the transmitter of a 16-QAM BCM-MSM-PTS OFDM system. First, we took two binary codes C_1 and C_2 to form a BCM QPSK sequence

$$Q = \{ (\frac{1}{\sqrt{2}} j^{C_{1,k}+2C_{2,k}}) e^{j\pi/4} : 0 \le k \le n-1 \}.$$

Then, we divide the QPSK sequence Q into M sub-blocks and transform these sub-blocks into the IFFT. And then, we use two binary codes C_3 and C_4 to construct the BCM codes for generating the phase rotation factor $\boldsymbol{B_i}$. We generate 16-QAM candidate sequences from the original QPSK modulation after linear combination multiplied by the phase rotation factor $\boldsymbol{B_i}$ as [1,j,-

1,-j] by MSM scheme. Finally, the transmitted OFDM signal with minimum PAPR is selected from these 16-QAM candidate sequences.

Obviously, the proposed BCM-MSM-PTS does not need to transmit side information to the receiver because each point in the 16-QAM constellation is the original QPSK modulation extension.

4. NUMERICAL RESULTS

This paper analyses the influence of several PAPR reduction methods on the PAPR of an 16-QAM OFDM system, where the PAPR reduction methods includes the PTS scheme, the M-PTS scheme, and the BCM-MSM-PTS techniques. For simulation, one hundred thousand data sequences where randomly chosen and number of subcarriers is N=1024 and 2048. Set the oversampling factor L=4 for selecting and estimating the signal with minimum PAPR. The proposed method uses four binary Reed-Muller codes to construct a 16-QAM BCM codes. In addition, this paper analyses the influence of Reed-Muller codes with three ordering are random ordering, nature ordering, and cyclic ordering. Note that the notation RMG(r,m) and CRM(r,m) and RRM(r,m) respectively are denoted as the rth order Reed-Muller codes of length 2^m with natural ordering and cyclic ordering and random ordering. The number of data bits $k = 1 + \binom{m}{1} + \cdots + \binom{m}{r}$, the minimum squared Euclidean distance $d_{\min} = 2^{m-r}$. The number of detection $d_{\min} \geq 2t$, and the number of correction can be written as $d_{\min} \geq 2t+1$, then $t = \left\lfloor \frac{d_{\min} - 1}{2} \right\rfloor$ expresses the correct bits, where $\lfloor \ \rfloor$ means the largest integer number no greater than $\frac{d_{\min} - 1}{2}$ [12].

This paper denotes that the number of sub-blocks is M=4 and the phase rotation factor of the PTS scheme is [1, -1, j, -j].

As shown in Fig. 4, when $CCDF = 10^{-4}$, the number of subcarriers N=1024, with random partition, PAPR = 8.7723 dB for the M-PTS scheme, PAPR = 8.7881 dB for the 16-QAM PTS, and PAPR = 8.8172 dB for the proposed BCM-MSM-PTS scheme that the data bits k =120, 210 and the correct bits t = 63, 31, and with adjacent partition, PAPR = 9.3799 dB for the M-PTS scheme, PAPR = 9.4037 dB for the 16-QAM PTS, and PAPR = 9.4045 dB for the proposed BCM-MSM-PTS that the data bits k =45, 120 and the correct bits t = 127, 63.

As shown in Fig. 5, when $CCDF = 10^{-4}$, the number of subcarriers N=2048, with random partition, PAPR = 9.1519 dB for the M-PTS scheme, PAPR = 9.1175 dB for the 16-QAM PTS, and PAPR = 9.1796 dB for the proposed BCM-MSM-PTS scheme that the data bits k =165, 330 and the correct bits t = 127, 63, and with adjacent partition, PAPR = 9.6052 dB for the M-PTS scheme, PAPR = 9.6422 dB for the 16-QAM PTS, and PAPR = 9.6797 dB for the proposed BCM-MSM-PTS that the data bits k =55, 165 and the correct bits t = 255, 127.

Obviously, even the ability of the PAPR reduction using M-PTS is better than the 16-QAM PTS and BCM-MSM-PTS, but it does not have any error correction capability to against the channel interference. To improve the error correction capability of the M-PTS scheme, our proposed method BCM-MSM-PTS has error correction capability and improves the PAPR performance effectively. Compared with the 16-QAM PTS scheme, the proposed method BCM-MSM-PTS has error correction capability and it also eliminates the need to transmit side information.

5. CONCLUSIONS

To improve the error correction capability of the M-PTS scheme is investigated in this paper. The proposed method, called BCM-MSM-PTS, combines the 16-QAM BCM codes with the M-PTS scheme. Although the M-PTS scheme has better PAPR performance than the proposed method, the signal transmitted by the M-PTS scheme does not have error correction capability. Compared with the 16-QAM PTS scheme, the proposed method has error correction capability and eliminates need to transmit the side information. This paper further provides the best selections of QPSK BCM codes to improve the error correction capability and the PAPR performance of 16-QAM OFDM signal.

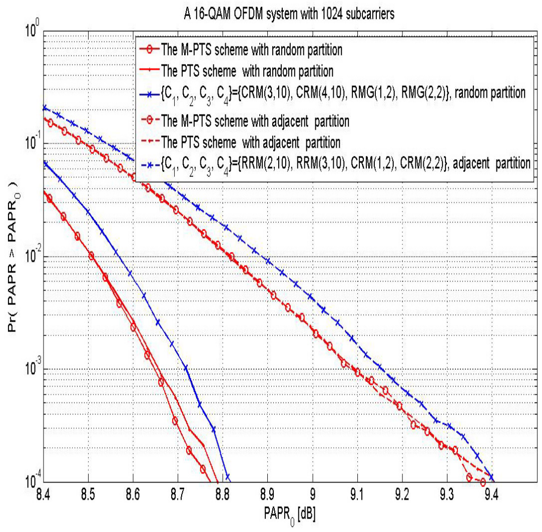

Fig. 4 Comparisons of CCDFs based on three PAPR reduction methods.

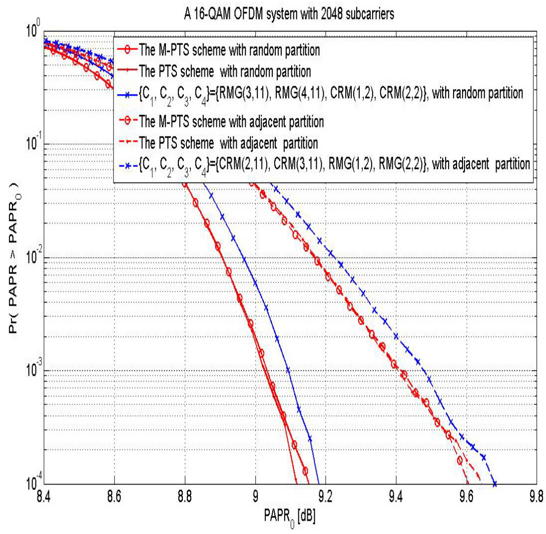

Fig. 5 Comparisons of CCDF based on three PAPR reduction methods.

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