Is Complex-Valued Arithmetic a Must for Equalizers in Two-Dimensional Modulations?

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***Abstract*— In this paper, the possibility of using real-valued arithmetic for equalizers in two-dimensional modulations is addressed. For two-dimensional modulations with coherent demodulation, the two sinusoidal signals used in the receiver for demodulating the band-pass signal are synchronized in phase to that of the signal received. This means that only amplitude distortion needs to be fixed and has nothing to do with the phase. Accordingly, the use of equalizers with complex-valued arithmetic can be avoided in this regard. Instead, using a pair of real-valued equalizers, one for the inphase and the other for the quadrature part of the two-dimensional signal is adequate. In this way, 50% reduction of the computations can be achieved compared to its complex-valued counterpart. Simulation results indicate that the real-valued approach performs even better than the conventional one.**

Keywords—Coherent demodulation, complex-valued equalizer, real-valued equalizer, computational complexity, symbol-error rate

1. INTRODUCTION

As equalizers for digital modulations with two-dimensional signals, schemes with complex-valued arithmetic are almost deemed as a must in the past. Does this be true? The answer is no. In this paper, the possibility of using real-valued arithmetic in this area of applications is addressed. Basic idea behind this is due to the fact that the two sinusoidal signals used for demodulating the two-dimensional signal is synchronized in phase to that of the signal received. This in effect restores the phase of the received signal back to its original. Therefore, the use of equalizers with complex-valued arithmetic for restoring the phase is no more necessary.

Traditionally, different forms of complex-valued equalizers (CVEs), for example [1-9], were used in two-dimensional modulation schemes. Approaches based on real-valued arithmetic have rarely found in literature. Equalizers with real-valued arithmetic in this field of applications can be found in [10, 11]. In [10], a quadrature amplitude modulation (QAM) signal equalizer without performing complex-valued training algorithms was presented. In [11], an approach based on transforming a complex-valued multiplication into three real-valued multiplications was proposed. Although only real-valued arithmetic is required in the approaches given in [10, 11], they didn't take advantage of the phase property in two-dimensional modulations with coherent demodulation. Using the property of phase synchronization for coherent demodulation as mentioned above, the intersymbol interference (ISI) encountered in the inphase and quadrature components can be treated independently and, hence, the use of equalizer with complex-valued arithmetic can be avoided. The new approach consists of two real-valued equalizers (RVEs), one for the inphase and the other for the quadrature component of the two-dimensional signal, from which much more efficient in computation is made possible.

Symbol-error rate (SER) performance for the new approach and the one using complex-valued arithmetic is compared in this paper. Results show that the new approach is equal to or even better than that of the conventional one.

2. REAL-VALUED EQUALIZERS FOR TWO-DIMENSIONAL MODULATIONS

Signals that can be described in terms of two orthogonal basis functions are treated as two-dimensional signals. This is the case for *M*-ary PSK and *M*-ary QAM modulation schemes. A two-dimensional signal can be written in the form

$$s(t)=a_1(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t+\varphi)$$
$$-a_2(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t+\varphi) \tag{1}$$

where $a_1(t)$ and $-a_2(t)$ are, respectively, the inphase and quadrature components, representing by pulses of duration equal to the symbol interval T , f_c is the carrier frequency, and φ is the initial phase of the carrier.

Fig. 1 shows a block diagram for a general band-pass signal transmitter. The resulting band-pass signal is transmitted through a communication channel to the receiver. For coherent demodulation the signal is demodulated by using a pair of the sinusoidal signals synchronized in phase to that of the signal received. The demodulation operation transfers the received band-pass signal back to a two-dimensional signal. Fig. 2 shows a band-pass signal receiver, where the phase of the two sinusoidal signals is assumed to be the same as that of in the transmitter. After product modulating, a pair of low-pass filters (LPFs) is used to produce the base-band components, called $x_I(t)$ and $x_Q(t)$ in Fig. 2. In response to the transmitted signal given in (1), the received signal can be of the form

$$r(t)=\tilde{a}_1(t)\sqrt{\frac{2}{T}}\cos(2\pi f_c t+\varphi+\theta)$$
$$-\tilde{a}_2(t)\sqrt{\frac{2}{T}}\sin(2\pi f_c t+\varphi+\theta) \tag{2}$$

where $\tilde{a}_1(t)$ and $\tilde{a}_2(t)$ are, respectively, the distorted versions of $a_1(t)$ and $a_2(t)$ and θ is the phase shift caused by the communication channel. In this case, a loss of phase synchronization in the process of demodulation leads to a phase rotation on the two-dimensional signal. For instance, if the received signal in (2) is demodulated by using a pair of sinusoidal signals as indicated in Fig. 2, the output of the demodulator for the inphase part, denoted as $x_{I,d}(t)$, will be

$$x_{I,d}(t)=\frac{\tilde{a}_1(t)}{T}\{\cos\theta+\cos(4\pi f_c t+2\varphi+\theta)\}$$
$$-\frac{\tilde{a}_2(t)}{T}\{\sin\theta+\sin(4\pi f_c t+2\varphi+\theta)\} \tag{3}$$

and the output of the demodulator for the quadrature part, denoted as $x_{Q,d}(t)$, will be

$$x_{Q,d}(t)=-\frac{\tilde{a}_1(t)}{T}\{\sin\theta-\sin(4\pi f_c t+2\varphi+\theta)\}$$
$$-\frac{\tilde{a}_2(t)}{T}\{\cos\theta-\cos(4\pi f_c t+2\varphi+\theta)\} \tag{4}$$

After low-pass filtering, the two base-band components become

$$x_I(t)=\frac{\tilde{a}_1(t)}{T}\cos\theta-\frac{\tilde{a}_2(t)}{T}\sin\theta \tag{5}$$

and

$$x_Q(t)=-\frac{\tilde{a}_1(t)}{T}\sin\theta-\frac{\tilde{a}_2(t)}{T}\cos\theta \tag{6}$$

Fig. 3 shows the effect of θ on the signal constellation. In the figure shown, a phase change from 0° to 20° is given and $(\tilde{a}_1(t)/T,\tilde{a}_2(t)/T)=(\pm1,\pm1)$ is assumed. It is obviously from (5) and (6) that the inphase and quadrature components are interrelated by a nonzero value of θ. In this case, it is necessary to rotate the phase back to its original for receiving the signal. Note from (5) and (6) that, for $\theta=0$, the two base-band components become

$$x_I(t)=\frac{\tilde{a}_1(t)}{T} \tag{7}$$

and

$$x_Q(t)=-\frac{\tilde{a}_2(t)}{T} \tag{8}$$

That is, the two components now become independent in this case. This is exactly the case for two-dimensional modulations with coherent demodulation due to that fact that the phase rotation caused by the communication channel is automatically solved by a proper operation of coherent demodulation in the receiver.

Denote the two-dimensional signal to be transmitted as $a(n) = a_I(n) + ja_Q(n)$. Also, let $x_I(n)$ and $x_Q(n)$ be the sampled version of $x_I(t)$ and $x_Q(t)$ respectively. To rotate the phases caused by nonzero values of θ back to their originals, a CVE with input $x(n) = x_I(n) + jx_Q(n)$ and target signal $t(n) = a_I(n-\Delta) + ja_Q(n-\Delta)$, where Δ is the delay chosen for the target to be estimated, is required. Having this, the LMS type of tap-weights updating equation is of the form

$$w_i(n+1) = w_i(n) + \mu e(n) x^*(n), \quad i = 0, 1, \ldots, N-1 \tag{9}$$

where μ is the step size controlling the rate of convergence of the algorithm, the asterisk * denotes the complex conjugate, and N is the number of tap-weights. This is the well-known complex LMS algorithm [12].

In the case of coherent demodulation, $x_I(n)$ and $x_Q(n)$ can be targeted, respectively, toward $a_I(n-\Delta)$ and $a_Q(n-\Delta)$ with proper values of delay Δ. Therefore, the CVE in Fig. 2 can be replaced by two RVEs, one with $x_I(n)$ as the input and $a_I(n-\Delta)$ as the target and the other with $x_Q(n)$ as the input and $a_Q(n-\Delta)$ as the target. Fig. 4 depicts the new equalizer structure. The outputs of the equalizers for the inphase and quadrature parts are now of the forms

$$y_I(n) = \sum_{i=0}^{N-1} w_{I,i}(n) x_I(n-i) \tag{10}$$

and

$$y_Q(n) = \sum_{i=0}^{N-1} w_{Q,i}(n) x_Q(n-i) \tag{11}$$

The error signals for two parts are then

$$e_I(n) = a_I(n-\Delta) - y_I(n) \tag{12}$$

and

$$e_Q(n) = a_Q(n-\Delta) - y_Q(n) \tag{13}$$

where Δ is again the delay chosen for the target. Then the LMS algorithm for updating the tap-weights becomes

$$w_{I,i}(n+1) = w_{I,i}(n) + \mu e_I(n) x_I(n), \quad i = 0, 1, \ldots, N-1 \tag{14}$$

and

$$w_{Q,i}(n+1) = w_{Q,i}(n) + \mu e_Q(n) x_Q(n), \quad i = 0, 1, \ldots, N-1 \tag{15}$$

where μ is again the step size controlling the rate of convergence of the algorithm.

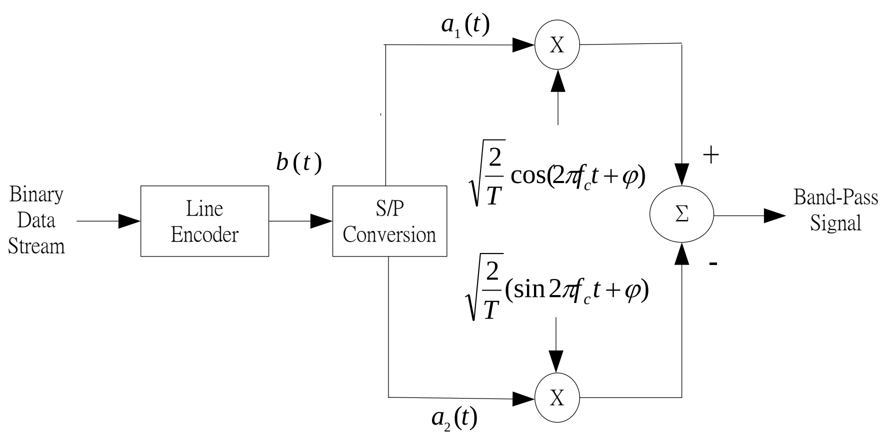


Fig. 1 Band-pass signal transmitter.

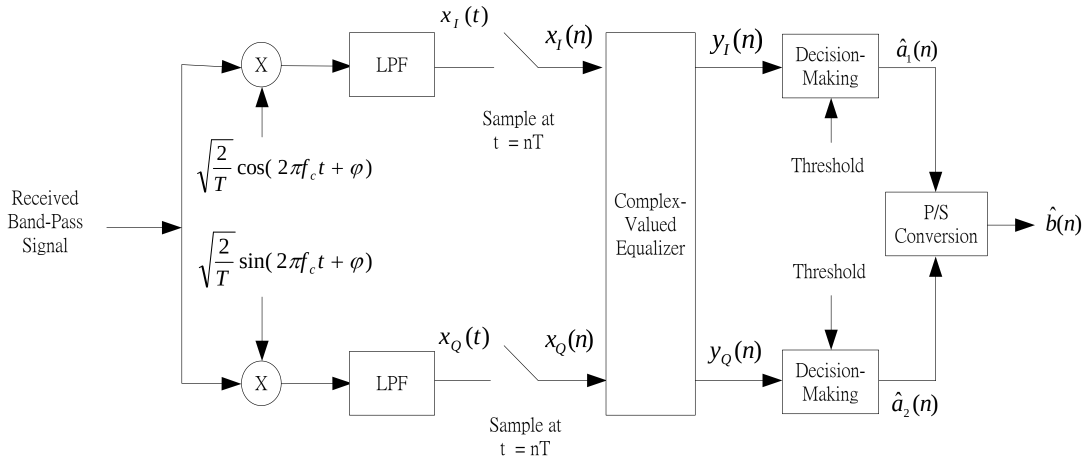


Fig. 2 Band-pass signal receiver with complex-valued equalizer inserted.

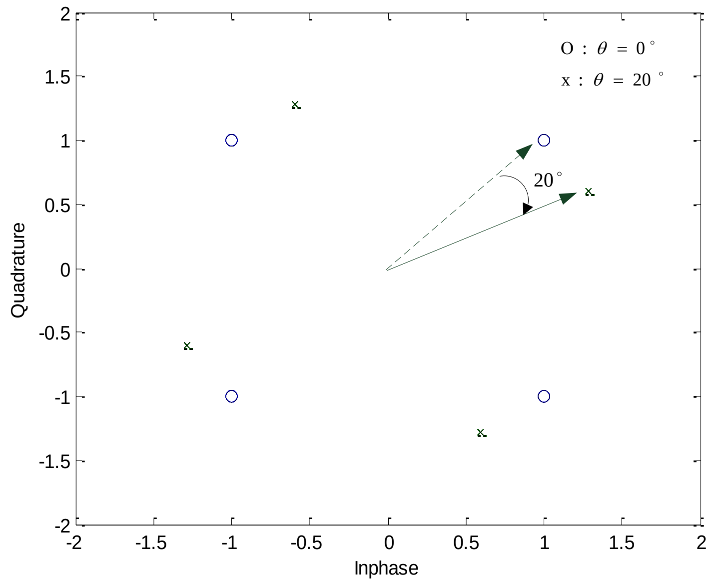


Fig. 3 Signal constellations with two different values of θ.

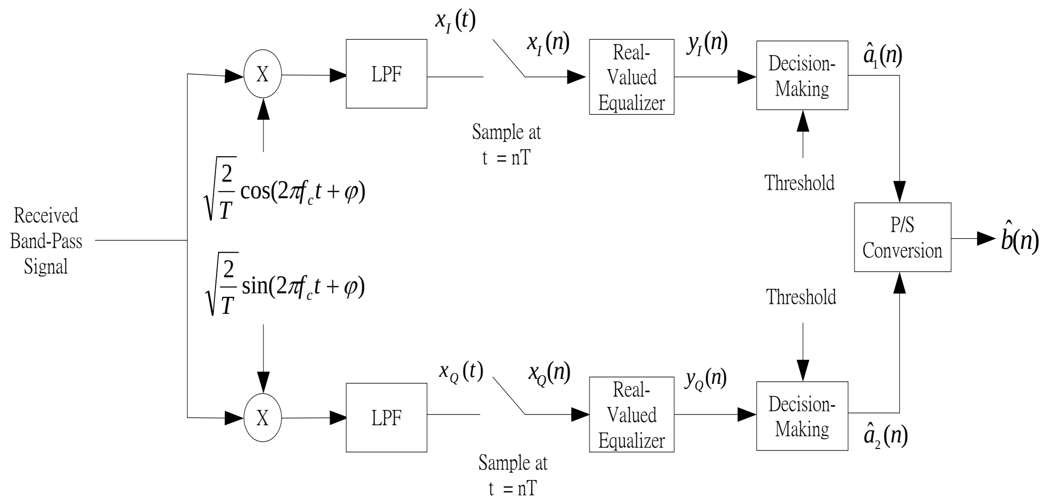


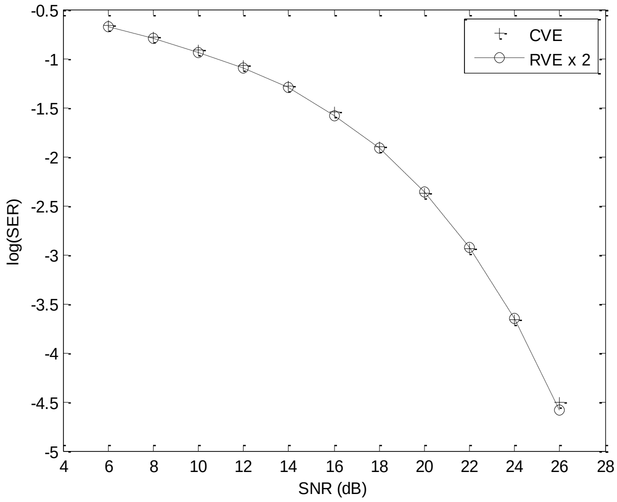
Fig. 4 Band-pass signal receiver with real-valued equalizers inserted.

3. COMPUTER SIMULATIONS

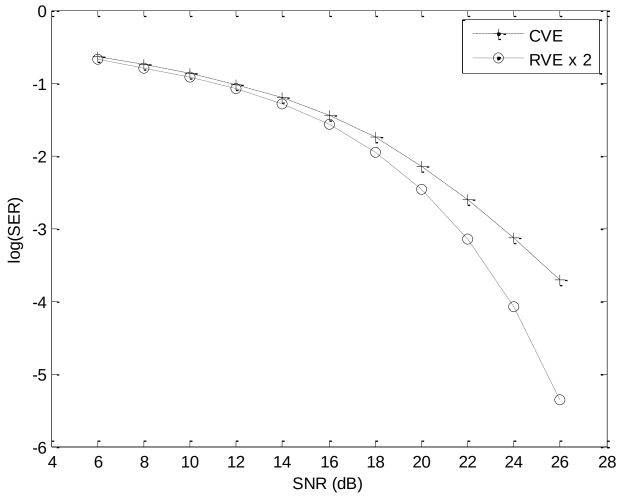
To justify the assertion made above, the two approaches are compared by using computer simulations. Fig. 5 depicts the block diagram used in the following simulations. A channel with real-valued parameters is assumed. This is the case for coherent demodulation due to the phase is kept the same as that of its original. In this simulation, the following channel is adopted:

$$H(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \qquad (16)$$

The input signal set chosen is $a(n) \in \{\pm 1 \pm j\}$. Related parameters used in the simulations are $N = 5$ and $\Delta = 1$. In addition, $\mu = 0.005$ and $\mu = 0.01$ are chosen for comparison. Fig. 6 shows the SER performance for the two approaches. It can be seen that the new approach performs the same or even better than the one using complex-valued arithmetic. The difference in performance could be inferred from the misadjustment caused in the adaptation process. It is well-known that larger values of μ and larger number of computations required in the adaptation result in a larger misadjustment [12]. Hence, the performance using the CVE may become worse than that of using the RVEs.



(a)



(b)

Fig. 6 SER performance vs. SNR for the two approaches under a perfect coherent demodulation with channel distortion determined by (16) with (a) $\mu = 0.005$ and (b) $\mu = 0.01$

4. CONCLUSIONS

As equalizers in two-dimensional modulations with coherent demodulation, a pair of RVEs, one for the inphase and the other for the quadrature component, is adequate without resort to any complex-valued arithmetic. Basic principle for this can be done and comparison of SER performance is given in this paper. A computational saving of 50% can be achieved compared to the complex-valued counterpart due to the fact that only real-valued arithmetic is required for the new approach. Computer simulation results show that the new approach has even better SER performance than that of conventional one.

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