# A Modified CES-PTS with Block Coded Modulation for PAPR Reduction in OFDM Systems

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*Abstract*—**High peak-to-average power ratio (PAPR), which increases the complexity of analog-to-digital (A/D) converter, is a major drawback of orthogonal frequency division multiplexing (OFDM) systems. Recently, the constellation extension scheme-partial transmit sequence (CES-PTS) scheme used in a 16-QAM OFDM system is a novel PAPR reduction method with low computational loads and has been proposed for improving the complexity of the constellation extension scheme (CES). Although the CES-PTS scheme has lower IFFT operations than the CES, the transmitted signal of the CES-PTS scheme has no error correction capability. Therefore, this paper proposes a modified CES-PTS scheme by combining block coded modulation (BCM) codes with the CES-PTS scheme for improving the error correction capability of the CES-PTS scheme, called BCM-CES-PTS. Moreover, we propose a new mapping of 16-QAM constellation extension scheme for improving the PAPR performance of the CES scheme. In simulation results, the BCM-CES-PTS scheme has better PAPR performance than the CES-PTS scheme when the number of sub-blocks is the same.**

*Keywords*— **OFDM, PAPR, BCM, BCM-PTS, CES-PTS**

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising transmission technique, since it has high spectral efficiency and low complexity receiver. Recently, OFDM technique has been proposed as the transmission technique used in a variety of wireless communication applications such as broadband wireless access standard HIPERMAN, 3GPP long term evolution (3GPP LTE) and IEEE 802.16x [1][2]. Although OFDM technique has many advantages, the transmitted OFDM signal has high peak-to-average power ratio (PAPR). The transmitted signal has high PAPR which results in the efficiency of high power amplifier to be reduced. Therefore, several PAPR reduction methods has been proposed such as partial transmit sequence (PTS) [3][4], constellation extension scheme [5] and CES-PTS [6].

The principle of CES-PTS is to partition the modulated sequence into several sub-blocks according to the position of symbols which have a pair of two possible constellation points. That is, the symbols which have no extended constellation points are included in the first sub-block; the other symbols are partitioned into several disjoint sub-blocks, where each sub-block whose elements have the same extended constellation points. Although the CES-PTS scheme has low complexity and no need to transmit side information in the transmitter, the transmitted signal of CES-PTS has no error correction capability. Therefore, this paper investigates how to combine the CES-PTS scheme with block coded modulation (BCM) codes to improve the error correction capability of CES-PTS.

Our proposed method, called BCM-CES-PTS, used in a 16-QAM OFDM system has better PAPR performance and error correction capability compared with CES-PTS. Moreover, our proposed method does not need to transmit side information in the transmitter.

## 2. OFDM SYSTEM MODEL

For an OFDM system with 16-QAM modulation, the OFDM baseband signal $x(t)$ in one symbol period can be written as

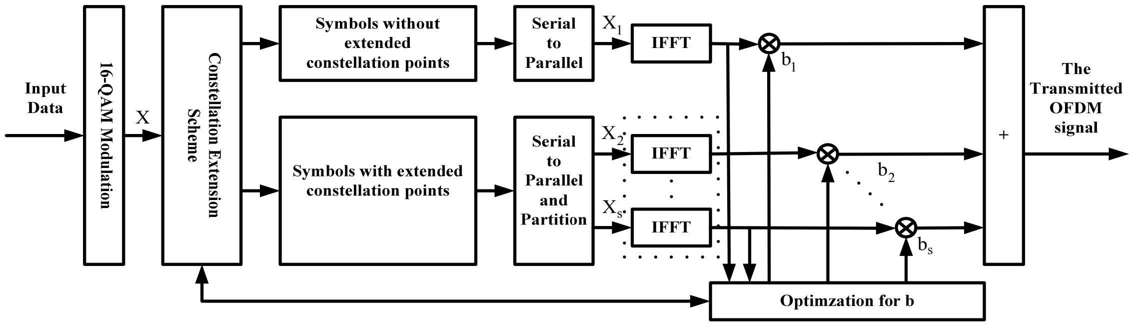


**Fig. 1 A block diagram of the CES-PTS system.**

$$x(t)=\sum_{n=1}^{N} S_n e^{j2\pi f_n t},\ 0 \le t \le T,$$

where $f_n$ is the frequency difference between subcarriers, $N$ is the number of subcarriers, $T$ is the OFDM symbol duration and $S_n$ is a 16-QAM symbol. The instantaneous power of $x(t)$ is defined as $P(t)=|x(t)|^2$ and the PAPR of the OFDM signal $x(t)$ is defined as

$$\text{PAPR}\left(x(t)\right)=\max_{0\le t\le T} P(t)\Big/E\left[|x(t)|^2\right],$$

where $E\left[|x(t)|^2\right]$ is the average power of $x(t)$ and $E[v]$ is the expected value of $v$.

For simulation proposes, the PAPR of the discrete time OFDM signal $x[n]$ can be evaluated by oversampling the continuous PAPR of $x(t)$ with the factor $L$. That is, the samples of OFDM signal are obtained by its discrete LN samples

$$x\left(\frac{iT}{LN}\right),\ i=0,1,\cdots LN-1.$$

This paper uses $L$=4 for numerical simulation, which is an acceptable approximation approved in the literature [2].

# 3. 16-QAM BCM CODES

Block coded modulation (BCM) appears at the very beginning of the coded modulation field and forms a class of high performance bandwidth efficient communication schemes [7]. In this paper, we focus on combining a 16-QAM BCM code with the CES-PTS scheme for improving the error correction capability of the CES-PTS scheme. In the 16-QAM BCM scheme, a coded sequence of $N$ symbols is obtained by using four binary codes of length $N$ as follows.

Let $C_1$, $C_2$, $C_3$ and $C_4$ to be a four binary codes of length $N$ with minimum Hamming distances $d_i$, $1 \le i \le 4$, respectively. Note that the minimum Hamming distance of each code needs to satisfy the condition of $d_1 < d_2 < d_3 < d_4$. A $4 \times N$ codeword array is constructed by filling the $i^{\text{th}}$ row with a codeword $c_i$ from the $i^{\text{th}}$ component code $C_i$:

$$\mathbf{c}=\begin{bmatrix}\mathbf{c}_1 \in C_1\\ \mathbf{c}_2 \in C_2\\ \mathbf{c}_3 \in C_3\\ \mathbf{c}_4 \in C_4\end{bmatrix}=\begin{bmatrix}c_{1,1} & c_{1,2} & \cdots & c_{1,N}\\ c_{2,1} & c_{2,2} & \cdots & c_{2,N}\\ c_{3,1} & c_{3,2} & \cdots & c_{3,N}\\ c_{4,1} & c_{4,2} & \cdots & c_{4,N}\end{bmatrix}.$$

Then, a complex-valued sequence $\mathbf{b}$ of length $N$ in a 16-QAM BCM code $\mathbf{b}=[b_0 b_1 \cdots b_{n-1}]$ can be formed by

$$b_k=\left(\frac{1}{\sqrt{2}} j^{c_{1,k}+2c_{2,k}}+\sqrt{2}\, j^{c_{3,k}+2c_{4,k}}\right)e^{j\frac{\pi}{4}},\ 0 \le k \le N-1. \quad (1)$$

# 4. THE CES-PTS SCHEME

Fig. 1 shows the block diagram of the CES-PTS scheme. The CES-PTS scheme first modulates the input data into the 16-QAM sequence $\mathbf{X}$, and then the 16-QAM sequence $\mathbf{X}$ is divided into several sub-blocks according to whether these symbols of $\mathbf{X}$ have a pair of two possible constellation points. Note that two possible constellation points are the extended constellation points and the original constellation points. More precisely, the CES-PTS scheme partitions the symbols which have no extended constellation points into the first sub-block, and then the symbols which have a pair of two possible constellation points are partitioned into the other sub-blocks. Assume that the number of extended constellation points used in the CES-PTS scheme is $s$.

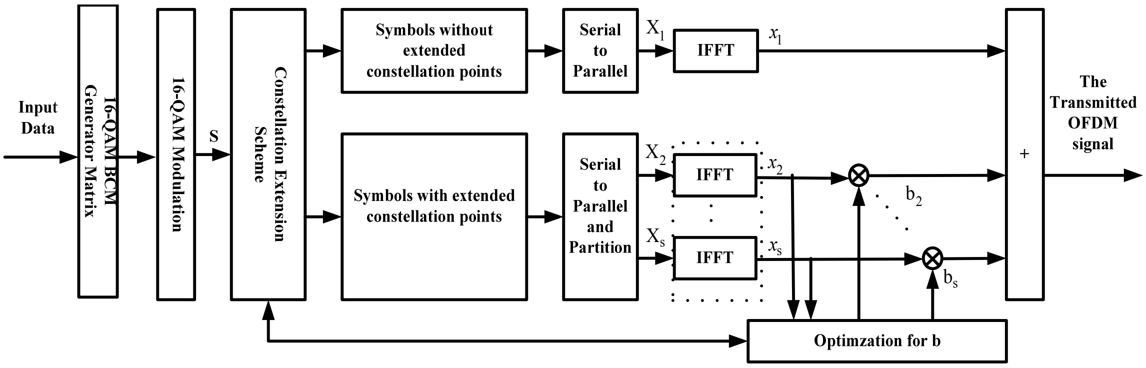


**Fig. 2 A block diagram of the BCM-CES-PTS system.**

Next, the partial transmit sequences can be formed by using the IFFT operations to transform these sub-blocks. The candidates, which are formed by multiplying the partial transmit sequences with the phase factor $\mathbf{b}$ = $\left[b_1\ b_2\ b_3\ \cdots\ b_s\right]$, are used for selecting the transmitted signal with minimum PAPR. In summary, the number of IFFT operations is s+1 and the number of candidates is $2^s$ if the phase factor $b_1 = 1$.

Compared with the CES, the CES-PTS scheme has low IFFT operations and is a suboptimum PAPR reduction method. However, the transmitted signal of the CES-PTS scheme has no error correction capability for improving PAPR performance in a 16-QAM OFDM system. Therefore, this paper proposes a modified CES-PTS scheme with error correction capability. The transmitter of our proposed method no needs to transmit the side information.

# 5. PROPOSED METHODS

Figure 2 shows the block diagram of the BCM-CES-PTS scheme. This paper proposes a modified CES based on the 16-QAM constellation with set partition. Assume that the number of extend constellation points used in the BCM-CES-PTS scheme is *s*.

The 16-QAM extension constellation of the BCM-CES-PTS scheme is shown in Fig. 3. First, the BCM-CES-PTS scheme encodes the information bits $\mathbf{u}$ into a 16-QAM BCM codeword by a 16-QAM BCM generator matrix $\mathbf{G}$ which can be written as

$$\mathbf{G} = \begin{bmatrix} \mathbf{G_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G_3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G_4} \end{bmatrix},$$

where $\mathbf{G}_1$, $\mathbf{G}_2$, $\mathbf{G}_3$ and $\mathbf{G}_4$ are the generator matrix of the code $C_1$, $C_2$, $C_3$ and $C_4$, respectively. More precisely, the 16-QAM codeword

$$\mathbf{uG} = \left[\mathbf{c_1} \quad \mathbf{c_2} \quad \mathbf{c_3} \quad \mathbf{c_4}\right]$$

can be formed into the complex-valued sequence $b_k$ by Eqn. (1).

This paper combines the BCM codes with the CES-PTS technique to improve the error correction capability of the CES-PTS technique. First, a 16-QAM BCM codeword is constructed by two QPSK BCM codewords and can be written as

$$b_k = \left\{ \frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} \right\} e^{j\left(\pi/4\right)}, 0 \le k \le N-1, \quad (2)$$

where $j=\sqrt{-1}$ and each component of two QPSK codewords can be denoted by $\{x_k, y_k\} \in \{0,1,2,3\}$.

For reducing the complexity of the CES-PTS scheme, this paper proposes a modified 16-QAM CES shown in Fig. 3(b). In the modified 16-QAM CES, the symbols without extend constellation points are denoted by

(1) $x_k = 0$ and $y_k = 1$,

(2) $x_k = 0$ and $y_k = 3$,

(3) $x_k = 1$ and $y_k = 0$,

(4) $x_k = 1$ and $y_k = 2$,

(5) $x_k = 3$ and $y_k = 0$,

(6) $x_k = 3$ and $y_k = 2$,

(7) $x_k = 2$ and $y_k = 1$,

(8) $x_k = 2$ and $y_k = 3$.

In addition, the symbols with extend constellation points can be classified into the following categories according to the difference in distance between the original constellation points with the extend constellation points:

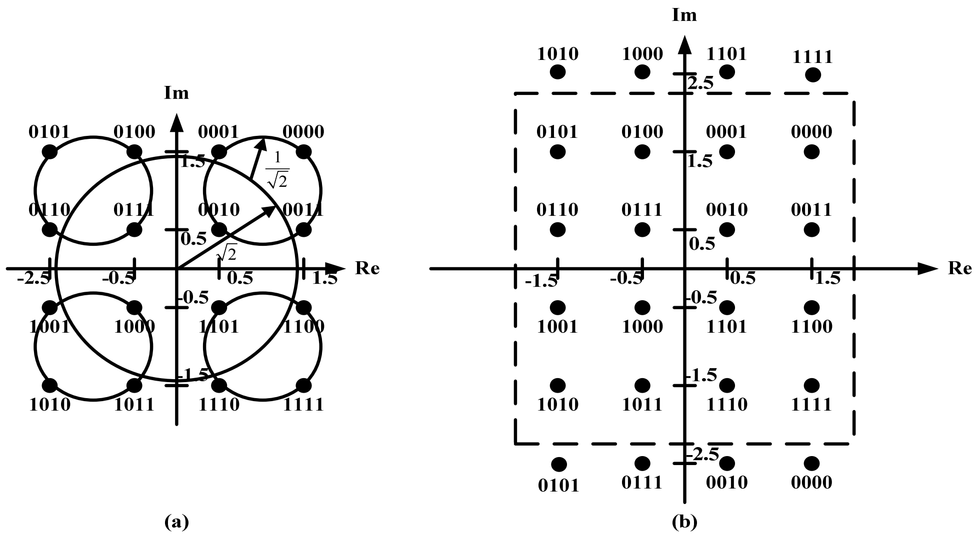


**Fig. 3: (a) a 16-QAM constellation with set partitioning. (b) an 16-QAM extension constellation with set partitioning.**

$$\begin{aligned} x(t) &= b_1 IFFT\{X_1\} + b_2 IFFT\{X_2\} + b_3 IFFT\{X_3\} + b_4 IFFT\{X_4\} + b_5 IFFT\{X_5\} \\ &= b_1 IFFT\{X_1\} + IFFT\{X_2 + b_2 P_2\} + IFFT\{X_3 + b_3 P_3\} + IFFT\{X_4 + b_4 P_4\} + IFFT\{X_5 + b_5 P_5\} \\ &= b_1 IFFT\{X_1\} + IFFT\{X_2\} + b_2 IFFT\{P_2\} + IFFT\{X_3\} + b_3 IFFT\{P_3\} + IFFT\{X_4\} + b_4 IFFT\{P_4\} \\ &\quad + IFFT\{X_5\} + b_5 IFFT\{P_5\} \\ &= \{b_1 IFFT\{X_1\} + IFFT\{X_2\} + IFFT\{X_3\} + IFFT\{X_4\} + IFFT\{X_5\}\} \\ &\quad + \{b_2 IFFT\{P_2\} + b_3 IFFT\{P_3\} + b_4 IFFT\{P_4\} + b_5 IFFT\{P_5\}\} \\ &= \{b_1 IFFT\{X_1\} + IFFT\{X_2\} + IFFT\{X_3\} + IFFT\{X_4\} + IFFT\{X_5\}\} \\ &\quad + IFFT\{b_2 P_2 + b_3 P_3 + b_4 P_4 + b_5 P_5\} \end{aligned} \quad (3)$$

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1. $\{x_k = 1, y_k = 1\}$ and $\{x_k = 0, y_k = 0\}$ : the difference in distance between the original constellation points with the extend constellation points is $p_2 = -j4$.
2. $\{x_k = 2, y_k = 2\}$ and $\{x_k = 3, y_k = 3\}$ : the difference in distance between the original constellation points with the extend constellation points is $p_3 = +j4$.
3. $\{x_k = 1, y_k = 3\}$ and $\{x_k = 0, y_k = 2\}$ : the difference in distance between the original constellation points with the extend constellation points is $p_4 = -j3$.
4. $\{x_k = 2, y_k = 0\}$ and $\{x_k = 3, y_k = 1\}$ : the difference in distance between the original constellation points with the extend constellation points is $p_5 = +j3$.

Next, the BCM codeword is partitioned into several sub-blocks, where all symbols of the first sub-block have no extend constellation points. In addition, the symbols which have the same difference in distance between the original constellation points with the extended constellation points are in the same sub-block. Therefore, the maximum number of sub-blocks in our proposed method is at most 5 in a OFDM symbol duration *T*. Assume that the input BCM codeword are divided into 5 sub-blocks $\{X_1, X_2, X_3, X_4, X_5\}$. The candidates in a 16-QAM OFDM system with the BCM-CES-PTS scheme can be written as Eqn. (3), where $b_i \in \{0,1\}$, $1 \le i \le 5$ and the elements of $P_i = \{p_{i,j}\}$, $1 \le j \le N$, are the difference in distance between the original constellation points with the extended constellation points. Note that the nonzero position of $P_i$ is the same as the nonzero position of $X_i$. The transmitted OFDM signal with minimum PAPR is selected from the candidates.

# 6. Simulations

This paper analyses the BCM-CES-PTS scheme influenced the PAPR of OFDM system and compared the PAPR performance of our proposed method with the PTS scheme. A ten thousand OFDM blocks are randomly chosen for simulation and set the oversampling factor $L$=4 for selecting and estimating the signal with minimum PAPR. The modified 16-QAM extension constellation of the BCM-CES-PTS scheme, shown in Fig. 3 (b), is used for simulation.

Fig. 4 and Fig. 5 show the complementary cumulative distribution function (CCDF) curves of a 16-QAM OFDM system with difference PAPR reduction methods. The number of subcarriers in a 16-QAM OFDM symbol duration is N=1024 or 2048. Note that the notation GRM(r,m) and CRM(r,m) respectively are denoted as the rth order Reed-Muller codes of length $2^m$ with natural ordering and cyclic ordering.

In Fig. 4 and Fig. 5, we can see that the PAPR performance of the BCM-CES-PTS scheme with two ordering based on the same order of each component codes, where the PAPR performance of these two ordering is almost the same. In addition, the BCM-CES-PTS scheme has better PAPR performance than the CES-PTS scheme based on that the number of sub-blocks is at most 5 (that is, the number of candidates is 16 in both the BCM-CES-PTS scheme and CES-PTS scheme when $b_1$=1).

In Fig. 4 and Fig. 5, two ordering with four component sub-codes RM(r,m), $1 \le i \le 4$ .Which bit length is expressed as $n = 2^m$ , the number of data bits is $k = 1 + \binom{m}{1} + \cdots + \binom{m}{r}$ , the minimum squared Euclidean distance $d_{\min} = 2^{m-r}$ , number of detection $d_{\min} \ge 2t$ , correct number $d_{\min} \ge 2t+1, \quad t = \left\lfloor \frac{d_{\min}-1}{2} \right\rfloor$ .It can be seen. In Fig. 4 two ordering with four component sub-codes of bit length is 1024, the number of data bits k are 10, 45, 120, and 210, respectively. The minimum squared Euclidean distances $d_{\min}$ are 512, 256, 128, and 64, respectively. The detection number t are 256, 128, 64, and 32, respectively. The correct number t are 255, 127, 63, and 31, respectively. In Fig. 5 two ordering with four component sub-codes of bit length is 2048, the number of data bits k are 11, 55, 165, and 330, respectively. The minimum squared Euclidean distances $d_{\min}$ are 1024, 512, 256, and 128, respectively. The detection number t are 512, 256, 128, and 64, respectively. The number correct t are 511, 255, 127, and 63, respectively.

Although the PTS scheme has better PAPR performance than the BCM-CES-PTS scheme, the transmitted signal of the PTS scheme compared with the BCM-CES-PTS scheme has no error correction capability.

## 7. Conclusion

Since the CES scheme and PTS scheme have no error correction capability against channel effects, the proposed method improves the error correction capabilities of both the CES scheme and PTS scheme by combining a 16-QAM BCM code with the CES-PTS scheme. Our proposed method has at most 6 IFFT operations compared with the CES scheme with $2^8$ IFFT operations in a OFDM symbol duration $T$. Compared with the PTS scheme, the BCM-CES-PTS scheme does not need to transmit the side information in the transmitter.

## References


[1] IEEE 802.16d, “Air interface for fixed broadband wireless access systems,” IEEE 802.16, May 2004.

[2] T. Jiang and Y. Y. Wu, “An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals,” *IEEE Trans. Broadcast.*, vol. 54, no. 2, pp. 257-268, June 2008.

[3] L. J. Cimini Jr. and N. R. Sollenberger, “Peak-to-Average power ratio reduction of an OFDM signal using partial transmit sequences,” *IEEE Commun. Lett.*, vol. 4, no. 3, pp. 86-88, Mar. 2000.

[4] C. Tellambura, “Improved phase factor computation for the PAPR reduction of an OFDM signal using PTS,” *IEEE Commun. Lett.*, vol. 5, pp. 135-137, Apr. 2001.

[5] Y. J. Kou, W. -S. Lu and A. Antoniou, “A New Peak to Average Power Ratio Reduction Algorithm for OFDM Systems via Constellation Extension,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, May 2007.

[6] H. Liang, K. C. Chung, H. Chen and Y. F. Huang, “A Modified Constellation Extension Scheme with Low Complexity for PAPR Reduction in OFDM Systems,” *in Proc. IEEE WOCC*, pp. 1-4, May 2010.

[7] E. Cusack, “Error control codes for QAM signaling,” *Electron. Lett*, vol. 20, pp. 62-63, Jan. 1984.

[8] T.R.N. Rao, E. Fujiwara, “Error-control coding for computer systems,” Englewood Cliffs, N.J. Prentice Hall, 1989.

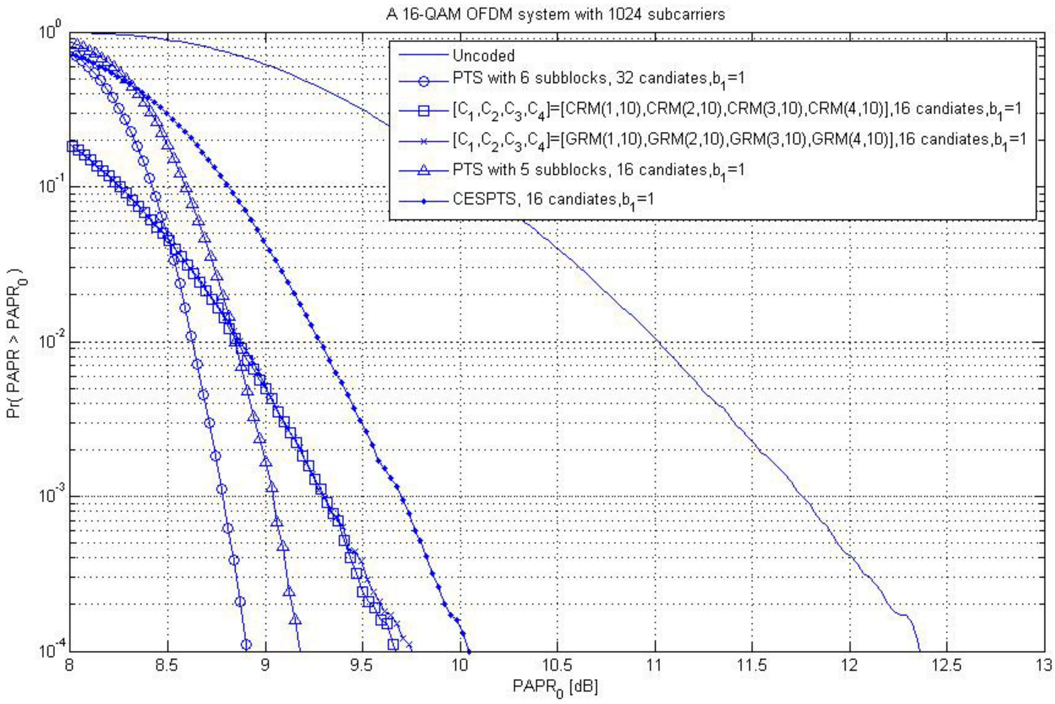


Fig.-4：Comparisons of CCDF based on different PAPR reductions.

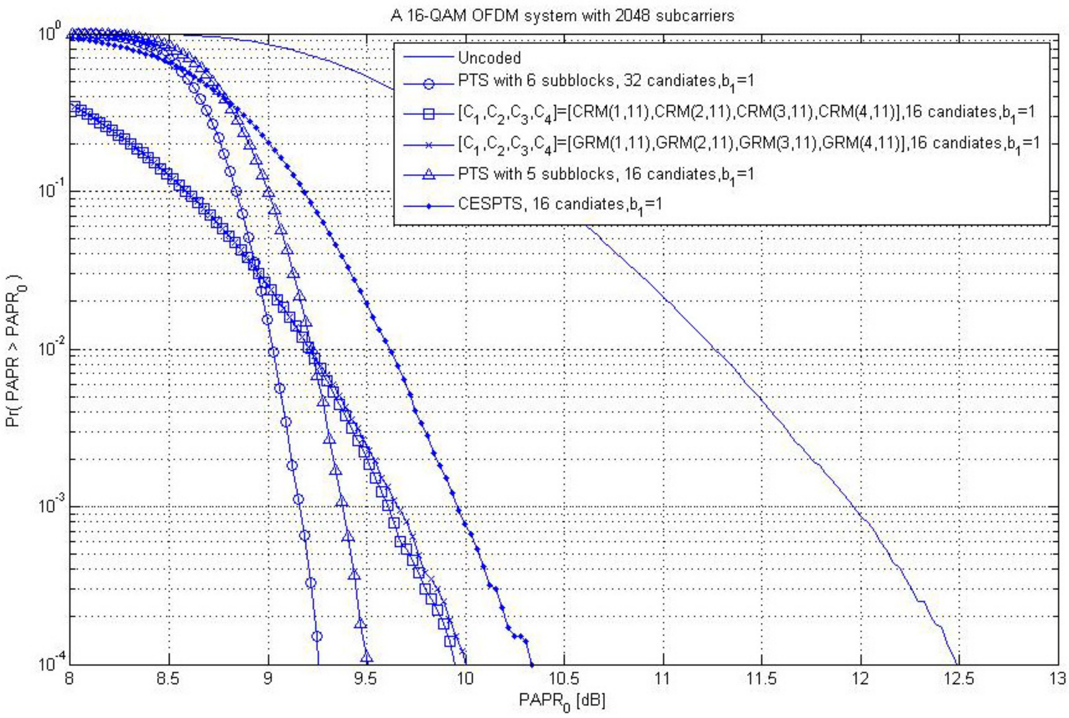


Fig.-5：Comparisons of CCDF based on different PAPR reductions.