Fuzzy Control of the DSP-based Pendubot at Mid Position

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Abstract

This paper gives a simple and natural control technique to realize the pendubot control. The switch controller is utilized. The fuzzy controller is exploited to stabilize the equilibrium point and the backwards and forwards energy pumping method is to swing up the pendulum. The fuzzy controller includes the fuzzy stabilizing controller and fuzzy target angle tuner. The backwards and forwards energy pumping method and the fuzzy target angle tuner are the main contributions of this paper.

1. Introduction

The pendubot (pendulum robot) is a two-link planar robot, which first link (arm) is actuated and second link (pendulum) rotates freely. The pendubot is a benchmark system for under-actuated robot manipulators, i.e., they having fewer actuators than degrees of freedom. The pendubot is also used for research in nonlinear control. Many studies have been proposed to swing up and balance the pendubot. Spong and Block [1] applied the partial feedback linearization approach to swing up the pendulum and used a linear quadratic regulator (LQR) to balance the pendulum. Fantoni, Lozano and Spong [2] utilized energy based control to swing up and balance the pendulum by simulations. Zhang and Tarn [5] proposed hybrid control to balance the pendulum. The hybrid control is composed of discrete-time control and continue-time control simultaneously. Sanchez *et al.* [4] realized balance control of the pendubot via fuzzy control. Combining linear regulator theory with Takagi-Sugeno fuzzy technology, the tracking control is implemented. Craig *et al*. [3] implemented the arm-driven inverted pendulum system as a mechatronic system design case study. The arm-driven inverted pendulum is the same as the pendubot. Development of the swing-up and balancing controller and development of the embedded controller for the arm-driven inverted pendulum system are still continuations of their work. From another viewpoint, Tanaka *et al.* [6] realized acrobatic game of the pendubot by simulations. The acrobatic game is keeping the arm swinging periodically, while the pendulum maintains standing vertically.

As the same as Spong and Block [1], the switch controller is utilized in this paper. The switch control (the self-erecting control) diagram is shown in Fig. 1. There are two control modes in this control system. At first, the swing-up control is applied and the pendulum is swung upward. When the pendulum is in the range of balancing control, the control mode is switched to the balance controller. In this paper, the fuzzy controller is exploited to stabilize the equilibrium point and the backwards and forwards energy pumping method is to swing up the pendulum. The fuzzy controller includes the fuzzy stabilizing controller and fuzzy target angle tuner. The backwards and forwards energy pumping method and the fuzzy target angle tuner are the main contributions of this paper.

The apparatus EMECS (Electro-Mechanical Engineering Control System) (Terasoft Inc. [10]; Li *et al.* [7,8]) used in this paper is supplied by TeraSoft Inc. The main feature of EMECS is an embedded control system. Because the digital controller of EMECS is the TMS320F2812 chip (Texas Instruments Inc. [11]), EMECS can operate stand-alone. The photograph of the DSP (digital signal processor)-based pendubot is shown in Fig. 2. The system schematic is shown in Fig. 3. The dc motor is driven by the PWM port of TMS320F2812 chip. The encoders of the dc motor and pendulum are linked to the QEP port of TMS320F2812 chip.

2. System description

Considering the pendubot schematic as shown in Fig. 4, the system variables and parameters are defined as follows:

m_a : mass of arm, m_p : mass of pendulum,

l_a : length of arm, c_a : center of gravity of arm,

c_p : center of gravity of pendulum,

J_a : inertia of arm,

J_p : inertia of pendulum,

θ_a : angular displacement of arm,

θ_p : angular displacement of pendulum,

g : gravitational acceleration,

τ_m : torque generated by the dc motor.

The dynamic equation of motion can be derived by the Euler-Lagrange equation (Li *et al.*, [7,8]) and is written as follows,

$$M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = \tau = \begin{bmatrix} \tau_m \\ 0 \end{bmatrix} \qquad (1)$$

where $q = \begin{bmatrix} \theta_a \\ \theta_p \end{bmatrix}$,

$$M(q) = \begin{bmatrix} P_1 + P_2 + 2P_3\cos\theta_p & P_2 + P_3\cos\theta_p \\ P_2 + P_3\cos\theta_p & P_2 \end{bmatrix},$$

$$B(q,\dot{q}) = \begin{bmatrix} -P_3\dot{\theta}_p\sin\theta_p & -P_3(\dot{\theta}_a + \dot{\theta}_p)\sin\theta_p \\ P_3\dot{\theta}_a\sin\theta_p & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} P_4\cos\theta_a + P_5\cos(\theta_a + \theta_p) \\ P_5\cos(\theta_a + \theta_p) \end{bmatrix}.$$

The parameters of matrices $M(q)$, $B(q,\dot{q})$ and $G(q)$ are $P_1 = J_a + m_a c_a^2 + m_p l_a^2$, $P_2 = J_p + m_p c_p^2$, $P_3 = m_p l_a c_p$, $P_4 = (m_a c_a + m_p l_a)g$, $P_5 = m_p c_p g$.

The torque generated by the dc motor is

$$\tau_m = \frac{k_i}{R_a}v_a - \frac{k_i k_b}{R_a}\dot{\theta}_a \qquad (2)$$

where R_a is the armature resistance, v_a is the applied voltage, k_i is the torque constant and k_b is the back-emf constant.

3. Backwards and forwards swing-up

In this section, the backwards and forwards swing-up method is proposed. Total energy of the pendubot is given by (Fantoni, Lozano and Spong, 2000)

$$E = \frac{1}{2}\dot{q}^T M(q)\dot{q} + P_4\sin\theta_a + P_5\sin(\theta_a + \theta_p) \qquad (3)$$

The rate of the total energy change is

$$\frac{dE}{dt} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T G(q)$$

$$= \dot{q}^T(-B(q,\dot{q})\dot{q} - G(q) + \tau) + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T G(q)$$

$$= \dot{q}^T\tau + \frac{1}{2}\dot{q}^T(\dot{M}(q) - 2B(q,\dot{q}))\dot{q}$$

$$= \dot{q}^T\tau = \dot{\theta}_a\tau_m \qquad (4)$$

Integrating both sides of (4), we can obtain

$$\int_{t1}^{t2} \dot{\theta}_a\tau_m dt = \bar{\tau}_m \cdot \Delta\theta_a = E(t2) - E(t1) \qquad (5)$$

where $\bar{\tau}_m$ is the average torque and $\Delta\theta_a$ is the arm angle from $t1$ to $t2$. Taking into account of Fig. 5 and assuming the kinetic energy of pendulum is zero, the arm is swung from $\theta_a = -\pi/2 + \theta_W$ to $\theta_a = -\pi/2 - \theta_W$. The energy $E(t2) - E(t1) > 0$ because $\Delta\theta_a = 2\theta_W$ and $\bar{\tau}_m > 0$. When the kinetic energy is transferred to the potential energy completely, the arm is being swung backward $\theta_a = -\pi/2 + \theta_W$. Then the potential energy of pendulum is accumulated little by little. Finally, the pendulum is swung-up to the range of balance control.

4. Fuzzy stabilization

The fuzzy stabilizing controller (Yen and Langari [9]) includes four parts, i.e., a fuzzification interface, a fuzzy rule base, a fuzzy inference engine and a defuzzification interface. The fuzzification interface maps a noisy measurement to fuzzy sets within certain input universes of discourse. The fuzzy inference engine infers fuzzy logic control actions from a rule base based on the expert experience. The defuzzification interface yields a deterministic control input from an inferred control action. Typical linguistic sets used in this section have the forms: PB (positive big), PM (positive middle), PS (positive small), ZE (zero), NS (negative small), NM (negative middle) and NB (negative big). The membership functions are shown in Fig. 6.

In this section, the knowledge pertaining to the control problem is formulated in terms of a set of fuzzy inference rules. The inference rule has the following fuzzy conditional propositions

$$\textbf{\textit{If}}\ \left(\eta_a\left(\theta_a+\theta_p\right),\eta_b\Delta\left(\theta_a+\theta_p\right)\right)\ \textbf{\textit{is}}\ A\times B,$$

$$\textbf{\textit{then}}\ v_a\ \textbf{\textit{is C}} \qquad (6)$$

where η_a and η_b is the normalized factors and $[A\times B]\left(\eta_a\left(\theta_a+\theta_p\right),\eta_b\Delta\left(\theta_a+\theta_p\right)\right)$

$=\min\left(\eta_a\left(\theta_a+\theta_p\right),\eta_b\Delta\left(\theta_a+\theta_p\right)\right)$ for all $\left(\theta_a+\theta_p\right)\in\left[72^O,108^O\right]$ and $\Delta\left(\theta_a+\theta_p\right)\in\left[-36^O,36^O\right]$.

The fuzzy numbers *A*, *B* and *C* are chosen from the set of fuzzy numbers that represent the linguistic states NB, NM, NS, ZE, PS, PM, and PB. The total number of possible non-conflicting fuzzy inference rules is 49. They can conveniently be represented in Table 1.

The centroid method is used as the defuzzification interface and calculates the weighted average of fuzzy set. The centroid defuzzification can be expressed by the following equation

$$v_a=\eta_c\times\frac{\sum_{n=1}^{49}C\left(\left(v_a\right)_n\right)\times\left(v_a\right)_n}{\sum_{n=1}^{49}C\left(\left(v_a\right)_n\right)} \qquad (7)$$

where η_c is the denormalized factor. The fuzzy set *C* is shown in Fig. 6. In this section, the normalized factors are $\eta_a=10/\pi$ and $\eta_b=10^6$ and the denormalized factor is $\eta_c=24$.

5. Fuzzy tuning of the target angle

Because only the angle sum of pendulum and arm $\left(\theta_a+\theta_p\right)$ is considered in Section 4, the arm angle can not be fixed after the pendulum has been swung-up. In this section, we adjust the target angle of the sum $\left(\theta_a+\theta_p\right)$ as shown in Fig. 7. Referring to Fig. 7(a), the arm angle is less than -90^o and the target angle of the sum is less than 90^o. In order to avoid falling of the pendulum, the arm is pulled to the desired angle ($\theta_a=-90^O$). Considering Fig. 7(b), the arm angle is more than -90^o and the target angle is more than 90^o. The arm is pushed to the desired angle again. Hence the arm is fixed at the desired angle. The inference rules have the following canonical forms

$$\textbf{\textit{If}}\ \eta_d\theta_a\ \textbf{\textit{is P, then}}\ \theta_T\ \textbf{\textit{is P}} \qquad (8)$$

$$\textbf{\textit{If}}\ \eta_d\theta_a\ \textbf{\textit{is Z, then}}\ \theta_T\ \textbf{\textit{is Z}} \qquad (9)$$

$$\textbf{\textit{If}}\ \eta_d\theta_a\ \textbf{\textit{is N, then}}\ \theta_T\ \textbf{\textit{is N}} \qquad (10)$$

where η_d is the normalized factor. Applying centroid defuzzification to a fuzzy conclusion can be expressed by

$$\theta_T=\eta_e\times\frac{\sum_{n=1}^{3}E\left(\left(\theta_T\right)_n\right)\times\left(\theta_T\right)_n}{\sum_{n=1}^{3}E\left(\left(\theta_T\right)_n\right)} \qquad (11)$$

where η_e is the denormalized factor. The fuzzy sets *D* and *E* are defined in Fig. 8. In this section, the normalized factor is $\eta_d=\pi/2$ and the denormalized factor is $\eta_e=0.05\,\pi$.

6. Experimental results and conclusions

The experimental result is shown in Fig. 9. The angles of the arm and the sum are presented. Referring to Fig. 9, the upper curve is the angle of the arm and the lower curve is the angle of the sum of arm and pendulum. The total time period is 10 seconds. Before 4 seconds, the pendulum is swung up. After the pendulum reaches the range of equilibrium, the fuzzy control is utilized to balance the pendulum and pushes the arm to equilibrium position.

This paper presents the DSP-based control of the pendubot. The EMECS is utilized as the system hardware. The CCS (code composer studio) platform is exploited for controller software. The controller is realized by C language. The backwards and forwards energy-based control is used to swing up the pendulum and the fuzzy control is used to balance the pendulum. The backwards and forwards swing-up and the fuzzy target angle tuner are novel ideas for this subject. This paper is also appropriate for the fuzzy control education.

7. References

[1] Spong, M. W. and D. J. Block, "The pendubot: a mechatronic system for control research and education", ***Proc. 34th IEEE Conference Decision and Control***, pp.555-556, 1995.

[2] Fantoni, I., R. Lozano, and M. W. Spong, "Energy based control of the pendubot", ***IEEE Transactions on Automatic Control***, Vol.45, No.4, pp.725-729, 2000.

[3] Awtar, S., N. King, T. Allen, I. Bang, M. Hagan, D. Skidmore, and K. Craig, "Inverted pendulum systems: rotary and arm-driven – a mechatronic system design case study", ***Mechatronics***, Vol.12, No.2, pp.357-370, 2002.

[4] Begovich, O., E. N. Sanchez, and M. Maldonado, "Takagi-Sugeno fuzzy scheme for real-time trajectory tracking of an underactuated robot", ***IEEE Transaction Control Systems Technology***, Vol.10, No.1, pp.14-20, 2002.

[5] Zhang, M. and T. J. Tarn, "Hybrid control of the pendubot", ***IEEE/ASME Transaction Mechatronics***, Vol.7, No.1, pp.79-86, 2002

[6] Li, W., K. Tanaka and H. O. Wang, "Acrobatic control of a pendubot", ***IEEE Transaction Fuzzy Systems***, Vol.12, No.4, pp.549-554, 2004.

[7] Li, J.-H., M.-F. Wu, and W.-C. Lin, "Linear quadratic regulation of the dsp-based pendubot", ***Proceedings of the 2007 Intelligent Systems Conference on Engineering Applications***, Tainan, Taiwan, 2007.

[8] Li, J.-H., and W.-C. Lin, "H∞ state feedback disturbance rejection of the dsp-based pendubot", ***Proceedings of the 2007 CACS International Automatic Control Conference***, Taichung, Taiwan, pp.988-993, 2007.

[9] Yen, J. and R. Langari, ***Fuzzy logic, intelligence, control and information***, Engle-wood Cliffs, NJ, Prentice Hall, 1999.

[10] Terasoft Inc. Taiwan. Available: www.terasoft.com.tw

[11] Texas Instruments Inc. Available: www.ti.com

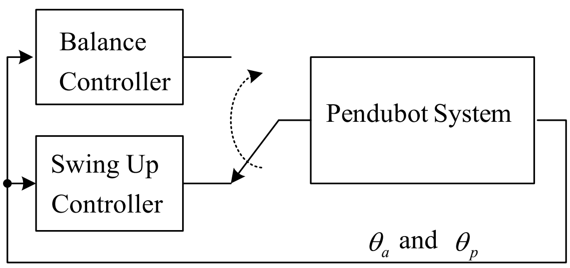


Fig. 1. The switch control diagram

Fig. 2. The DSP-based pendubot

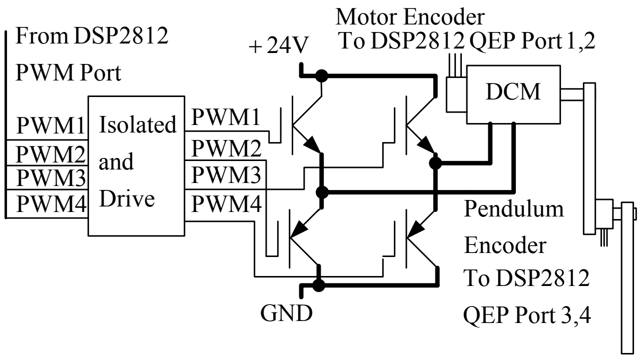


Fig. 3. The DSP-based pendubot schematic

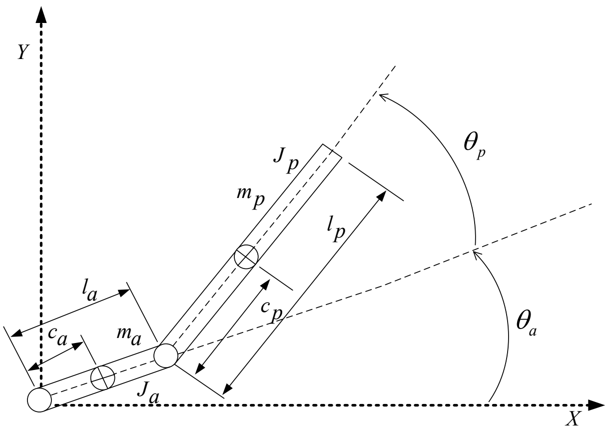


Fig. 4. The simplified pendubot schematic

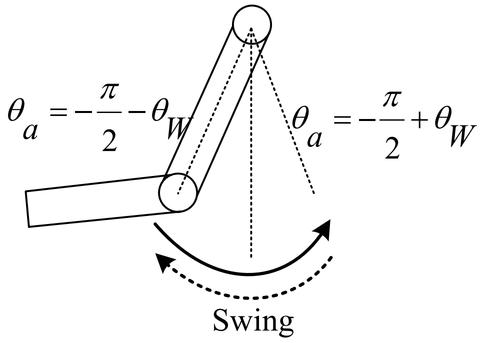


Fig. 5. Backwards and forwards pumping swing

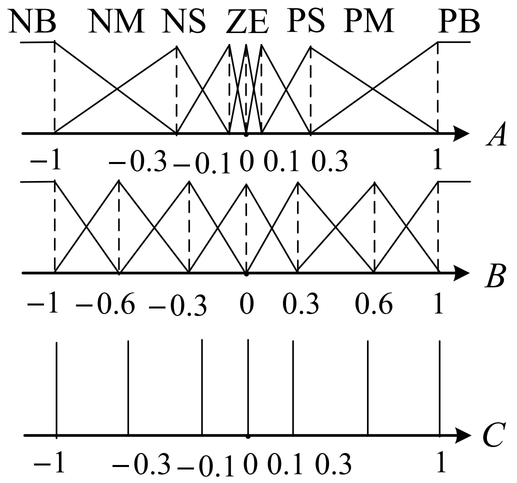
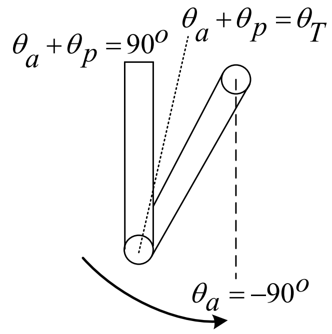
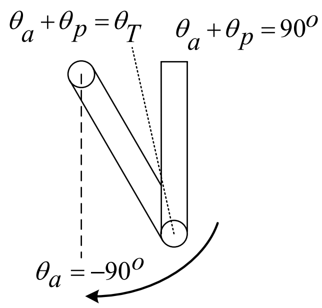


Fig. 6. The membership functions of *A*, *B* and *C*



7(a)



7(b)

Fig. 7. The fuzzy target angle tuning

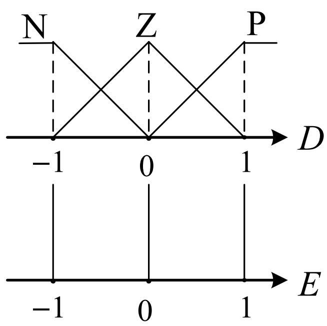


Fig. 8. The membership functions of *D* and *E*

Table 1. Rules of Fuzzy stabilizing Controller

v_a	$\eta_b\ \Delta(\theta_a+\theta_p)$							
$\eta_a(\theta_a+\theta_p)$		NB	NM	NS	ZE	PS	PM	PB
	NB	***PB***	***PB***	***PB***	***PM***	***PM***	***PS***	***ZE***
	NM	***PB***	***PM***	***PM***	***PM***	***PS***	***ZE***	***NS***
	NS	***PB***	***PM***	***PS***	***PS***	***ZE***	***NS***	***NM***
	ZE	***PM***	***PS***	***PS***	***ZE***	***NS***	***NS***	***NM***
	PS	***PM***	***PS***	***ZE***	***NS***	***NS***	***NM***	***NB***
	PM	***PS***	***ZE***	***NS***	***NM***	***NM***	***NM***	***NB***
	PB	***ZE***	***NS***	***NM***	***NM***	***NB***	***NB***	***NB***

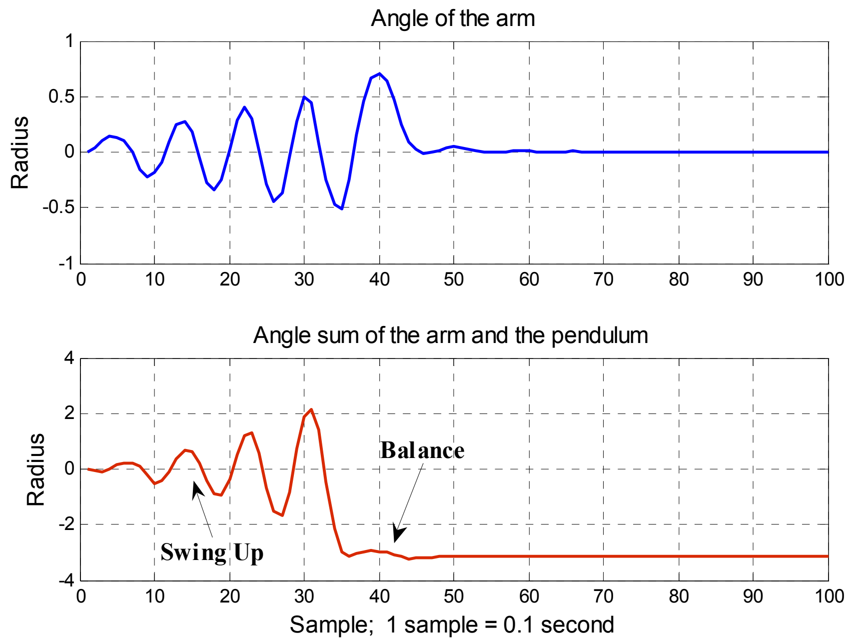


Fig. 9. The response of arm and sum of arm and pendulum