Correlation-Based Image Scaling with Separable and Nonseparable Kernels

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Abstract

Cubic convolution is a popular method for image interpolation. In the kernel of cubic convolution, there is an adjustable parameter whose value is -0.5. This value is not optimal for all images. In this paper, we propose a method that selects the parameter adaptively and apply cubic interpolation to image scaling. The original image is divided into non-overlapped blocks. The next step is to find the parameter suitable for each block and to scale up or down by the parameter. The final step is to combine the divided image to obtain the final image. Besides the separable kernels, we also use nonseparable kernels that have two adjustable parameters. Experiments indicate that the scaled images have better quality than those by fixed-parameter cubic convolution if optimal parameter is found. Using suboptimal parameters, the scaled images have compatible quality.

Keywords: correlation-based scaling, separable cubic interpolation, nonseparable interpolation

1. Introduction

As a result of the rapid development of computer as well as the universality of digital camera, digital image is acquired more easily than before. Compared with traditional analog pictures, digital images have more advantages. They can be effectively stored, transmitted and reprocessed. Owing to these reasons, digital images have been highly appreciated and applied in many ways.

Image scaling is a key operation for digital images. For example, the original image may be needed to enlarge to view in medical application and penal authenticate. Images may be also needed to be shrunk to fit the screen. Good algorithms of scaling will not loose details of images. Interpolation is a basic operation for image scaling as well as important methods for translation, rotation, and geometric correction. Interpolation is to calculate the values between known pixels in the original image by the weighting sum of neighbor pixels. The calculation can be represented by a convolution with certain functions that are called kernels.

Over the past few decades, a number of interpolation kernels have been devised for interpolation. These kernels may be separable or non-separable. The separable kernel $k(x,y)=k_1(x)k_2(y)$ can be expressed by a product of two one-dimensional (1-D) kernels. Among these 1-D kernels, nearest neighbor is the simplest one with strong aliasing and blurring effects. Linear interpolation is popular for hardware and software because it is easily realized [1], [7], [12]. Quadratic interpolation is disregarded largely due to phase distortion. However, quadratic interpolation generates images with quality close to cubic interpolation and takes much less time than cubic [3]. In piecewise nth-order polynomial kernels, cubic kernel provides good comprise between complexly and accuracy compared with kernels of 5th order, quintic and 7th order. Cubic kernel has significant efficiency so it has been applied to many applications [1], [2], [8]. In 2004, Blu, Tthévenaz, and Unser proposed linear kernel with time shift and obtained startling result [11]. Although separable kernels provide good performance for image scaling, digital images are not separable practically. Base on this reason, Reichenbach, etc. proposed non-separable kernels with multi-parameters constrained on biaxial symmetry, diagonal symmetry, continuity, DC-constant, interpolation, and smoothness [4], [5], [6], [9].

The structure of this paper is as following: Section 2 introduces the image scaling with interpolation kernels; Section 3 describes improved image scaling which with correlation-based parameters; Section 4 gives the result of experiment which with many real life images; and Section 5 is the conclusion and description of future work.

2. Image Scaling

Digital image processing is an important research subject for multimedia applications due to the versatility of digital images as well as rapid evolution of digital processors. After scaling, digital images can be displayed on screens of different resolutions or size. Fig. 1 shows a conceptual block diagram for which scaling operation is performed by reconstructing input discrete-time signal into a continuous-time signal, scaling it, and then re-sampling the scaled signal. The reconstruction can be expressed by

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n]\,k(t-nT) \qquad (1)$$

Where $x[n]$ is the 1-D discrete-time input signal, $x_c(t)$ is the reconstructed continuous-time signal, and $k(t)$ is the interpolation kernel. Scaling of continuous image is written as

$$x_c(at) = \sum_{n=-\infty}^{\infty} x[n]k(at-nT) \qquad (2)$$

Where a is the scaling factor, and sampler uses new period T' to sample $x_c(at)$. The output signal is

$$y[m] = x_c(amT') = \sum_{n=-\infty}^{\infty} x[n]k(amT'-nT) \qquad (3)$$

In (3), the reconstruction filter, time scaling, and sampler are combined into a scaling filter shown in Fig. 2 where

$$y[m] = x[n] * h[n] \qquad (4)$$

where $h[n] = k(nT)$ is a scaling filter.

Base on sampling theory, the function $\mathrm{sinc}(t)$ defined by

$$\mathrm{sinc}(t) \equiv \frac{\sin(\pi x)}{\pi x} \qquad (5)$$

is the interpolation kernel that interpolates band-limited signals perfectly if Nyquist criterion is satisfied. But this kernel has an infinite support and decays slowly such that it is impractical to use. Many other kernels such as linear interpolation, cubic B-spline, or cubic interpolation are proposed [12]. These kernels have excellent performance if they satisfy interpolation and DC-constant properties. The interpolation property of the kernel $k(x)$ can be expressed by

$$k(x) = \begin{cases} 1, & x = 0 \\ 0, & |x| = 1,2,\ldots \end{cases} \qquad (6)$$

Here zero crossing keeps the sample values of original digital image no changed if they are also sampled in new image. In order to ensure brightness of original digital image is not affected after scaling, the following property should be satisfied

$$\sum_{l=-\infty}^{\infty} k(d+l) = 1 \qquad (7)$$

for any value of d. These 1-D kernels can easily be used for image scaling in separable fashion where the horizontal and vertical directions are scaled independently.

$$y(u,v) = \sum_{m}\sum_{n} x[m,n]s(u-m, v-n) \qquad (8)$$

with 2-D kernel $s(u,v) = s_h(u)s_v(v)$ where $s_h(u)$ and $s_v(v)$ are two 1-D kernels.

3. Proposed Method

3.1 Fixed parameter kernels

The general separable piecewise *n*th-order polynomial kernels include nearest neighbor, linear, quadratic, and cubic convolution [3], [8], [12]. Cubic convolution is a popular method to image reconstruction because it has good compromise between complexity and accuracy. The kernel for cubic interpolation is

$$k_C(x)=\begin{cases}(\alpha+2)|x|^3-(\alpha+3)|x|^2+1, & 0\le|x|<1\\ \alpha|x|^3-5\alpha|x|^2+8\alpha|x|-4\alpha, & 1\le|x|<2\\ 0, & otherwise\end{cases} \tag{9}$$

where $\alpha=-0.5$ is fixed for all images [8]. But this value is not optimal for every image. In [10], Park pointed out if the frequency component of images is known, we can find the optimal value parameter α. To get closed to the characteristics of image, non-separable kernels with multi-parameters have been proposed. In [9], the simplest non-separable kernel with two parameters is proposed and expressed as

$$f(x,y)=\begin{cases}f_a(x,y) & 0\le|x|\le 1, 0\le|y|\le 1\\ f_b(x,y) & 1<|x|\le 2, 0\le|y|\le 1\\ f_c(x,y) & 1<|x|\le 2, 1<|y|\le 2\\ f_d(x,y) & 0\le|x|\le 1, 1<|y|\le 2\end{cases} \tag{10}$$

where

$$\begin{aligned}f_a(x,y)&=(x-1)(y-1)\\ &\times(xy+x+y+1+a_{02}\\ &\times(x^2y+xy^2+x^2+y^2)+a_{22}x^2y^2)\\ f_b(x,y)&=(x-1)(y-1)(x-2)^2\\ &\times((a_{02}+2)(y+1)+(2a_{02}+a_{22})y^2)\\ f_c(x,y)&=(x-1)(y-1)(x-2)^2(y-2)^2\\ &\times(4a_{02}+a_{22}+4)\\ f_d(x,y)&=(x-1)(y-1)(y-2)^2\\ &\times((a_{02}+2)(x+1)+(2a_{02}+a_{22})x^2)\end{aligned} \tag{11}$$

with $a_{02}=-(\alpha+2)$ and $a_{22}=\beta+(\alpha+2)^2$.

3.2 Correlation-Based parameters kernels

Since an image usually has various frequency components, it is insufficient for accuracy to fix the value of parameter. To make this claim clear, we perform the following experiment. The image "Lena" is down sampled by 2 and scaling up by 2 using the cubic convolution with various α. We calculate the PSNR between the original and resulting image. Fig. 3 shows the PSNR versus α in separable interpolation. It is obviously that there exists an optimal value at which the PSNR is maximal. It is also clear that the optimal values are distinct in different regions of the original images. In order to find the optimal value, we propose an improved method instead of keeping the value of α fixed. First, to reduce the complexity of computation, we divide the original image into many small blocks. Second, according to each block's characteristic we try to find the corresponding optimal parameter individually. Finally, we interpolate blocks respectively with individual optimal parameter and combine them into to the final image. As expected, we obtain more accuracy than the traditional one.

3.3 Test Patterns

One way to find the optimal value of α is to try every value possible values, to perform the scaling and calculate the PSNR, and then choose the optimal one. This approach is not practical due to complexity. In this paper we use a sub-optimal method based on correlation between the block and a set test patterns whose optimal values for scaling is pre-computed. Fig. 4 is the block diagram in which the divider divides original image into several blocks, the correlator computes the correlation between test patterns and image blocks to find optimal parameter for divided blocks. Scaling filter then scales each blocks with the optimal scaling factor, and finally, each scaled block are combined to form the output.

The test patterns are generated as following. First, we produce an image whose pixel values have identical independent Gaussian distribution with mean 32, 64, 96, 128, 160,192, 224 and variance 8, 16, 24, 32, 24, 16, 8 respectively. The probability density function is

$$f(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{12}$$

Where μ and σ^2 are mean and variance respectively. Second, these Gaussian patterns are filtered by low pass filters with circular-supported passbands of cut frequencies of $k\pi/8$ where $k=1,2,\cdots 8$ to obtained correlated Gaussian patterns. That is, the test patterns are formed by

$$c[m,n] = t[m,n] * h_{lp}[m,n] \tag{13}$$

where $t[m,n]$ is the image with identical independent Gausisan distribution, $c[m,n]$ is image with correlated distribution, and $h_{lp}[m,n]$ is the impulse response of the lowpass filter. The autocorrelation function $R[m,n]$ and power spectrum density $P(u,v)$ of $c[m,n]$ are expressed as:

$$R[m,n] = h_{lp}[m,n] \tag{14}$$

$$P(u,v) = \left|H_{lp}(u,v)\right|^2 = \begin{cases} 1, & \sqrt{u^2+v^2} \le \omega_c \\ 0, & others \end{cases} \tag{15}$$

where ω_c is the cut frequency. Fig. 5 shows the test patterns which have mean 128 and variance 32. Fig. 6 shows the magnitude spectrum and impulse response of the two-dimensional circular symmetric low pass filter with cut frequency $3\pi/8$.

The proposed method for image scaling is listed as following:

Setp 1. Divide input image $x[m,n]$ into image blocks $x_d[m,n]$ where $1 \le d \le K$ is the index of blocks and K is the number of blocks.

Setp 2. Compute the correlation γ_{dj} between $x_d[m,n]$ and test patterns $c_j[m,n]$, $1 \le j \le P$ where P is the number of test patterns.

Setp 3. Choose α_o that is the parameter associated with the test patterns that produce maximal γ_{dj}, $1 \le j \le P$.

Setp 4. Scale $x_d[m,n]$ into $y_d[m,n]$ with parameter α_o.

Setp 5. Combine $y_d[m,n]$ to obtain the output image $y[m,n]$.

The correlation coefficient γ_{dj} between $\mathbf{x}_d = x_d[m,n]$ and $\mathbf{c}_j = c_j[m,n]$ is computed by

$$\gamma_{dj} = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}(\mathbf{x}_d - \bar{x})(\mathbf{c}_j - \bar{c})}{\sqrt{\left(\sum_{m=1}^{M}\sum_{n=1}^{N}(\mathbf{x}_d - \bar{x})^2\right)\left(\sum_{m=1}^{M}\sum_{n=1}^{N}(\mathbf{c}_j - \bar{c})^2\right)}} \tag{16}$$

where $\bar{x}$, $\bar{c}$ are the mean value of $x_d[m,n]$ and $c_j[m,n]$:

$$\bar{x} = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N} x_d[m,n] \tag{17}$$

4. Result of Experiments

This section discusses the performance of the proposed method. Fig. 7 shows test images include Baboon, Lena, Jet, Moon, Viewer, Lake, Room, and Peppers that are standard test image for image processing.

First, we divide images to 4, 16, 64 blocks respectively to find the optimal parameter of every block, and utilize them to simulate. Second, we use test patterns instead. The original 256×256 image is down sampled by 2×2 to generate a 128×128 image, and then scaled up by 2×2 using one and two correlation-based parameters cubic and fixed parameter cubic kernel. To estimate the performance of scaled image, we consider Peak Image to Noise Ratio（PSNR）between the original image and the scaled one.

$$PSNR(dB) = 10\log_{10}\frac{255}{\sqrt{\frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}(x_{mn} - x'_{mn})^2}} \tag{18}$$

where x and x' are original and scaled image respectively.

TABLE I lists the PSNRs of fixed-parameter cubic convolution, optimal parameter cubic convolution, and correlation-based method. We can find that the PSNR of optimal parameter is higher than the fixed-parameter interpolation. This suggests that there is chance to obtain better result by choosing good parameter. However, correlation-based method has poor performance for most images than fixed-parameter interpolation. Only "Moon", "Viewer" and "Lake" have better or equal PSNRs. The reason of better performance is that the image blocks of "Moon", "Viewer" and "Lake" are like the correlated Gaussian noise such that correlation-based method also find the optimal parameter. In summary, the performance of correlation-based method highly depends on the test images.

Fig. 10 shows the results of 256×256 images scaled up by a factor of 1.5×1.5. Fig. 10 shows the results of 256×256 images scaled down by a factor of 0.5×0.5.

5. Conclusion

In this paper, we propose an correlation-based method for image scaling. In stead of using a fixed parameter in cubic convolution for interpolation, we try to find the better parameters for image blocks. The search of parameter is based on correlation of the block to some pre-computed test patterns whose optimal scaling parameter is known. We use the scaling parameter of the test pattern that has the largest correlation value to scale the original image. Correlation-based method performs poor for some images because our noise-like test patterns are not suitable for certain images. Detailed study and careful choice of test patterns should be carried out in future research. It is expected that the method could be applied to other image manipulation such as rotation or deformation.

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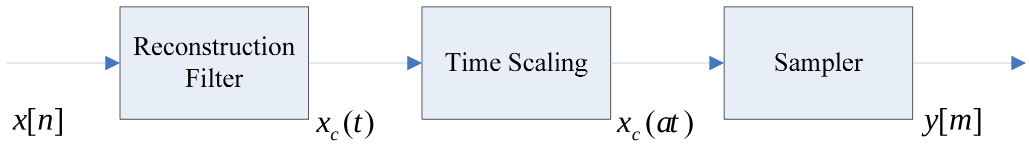


Fig. 1. The block diagram of image scaling.

Fig. 2. Scaling filter.

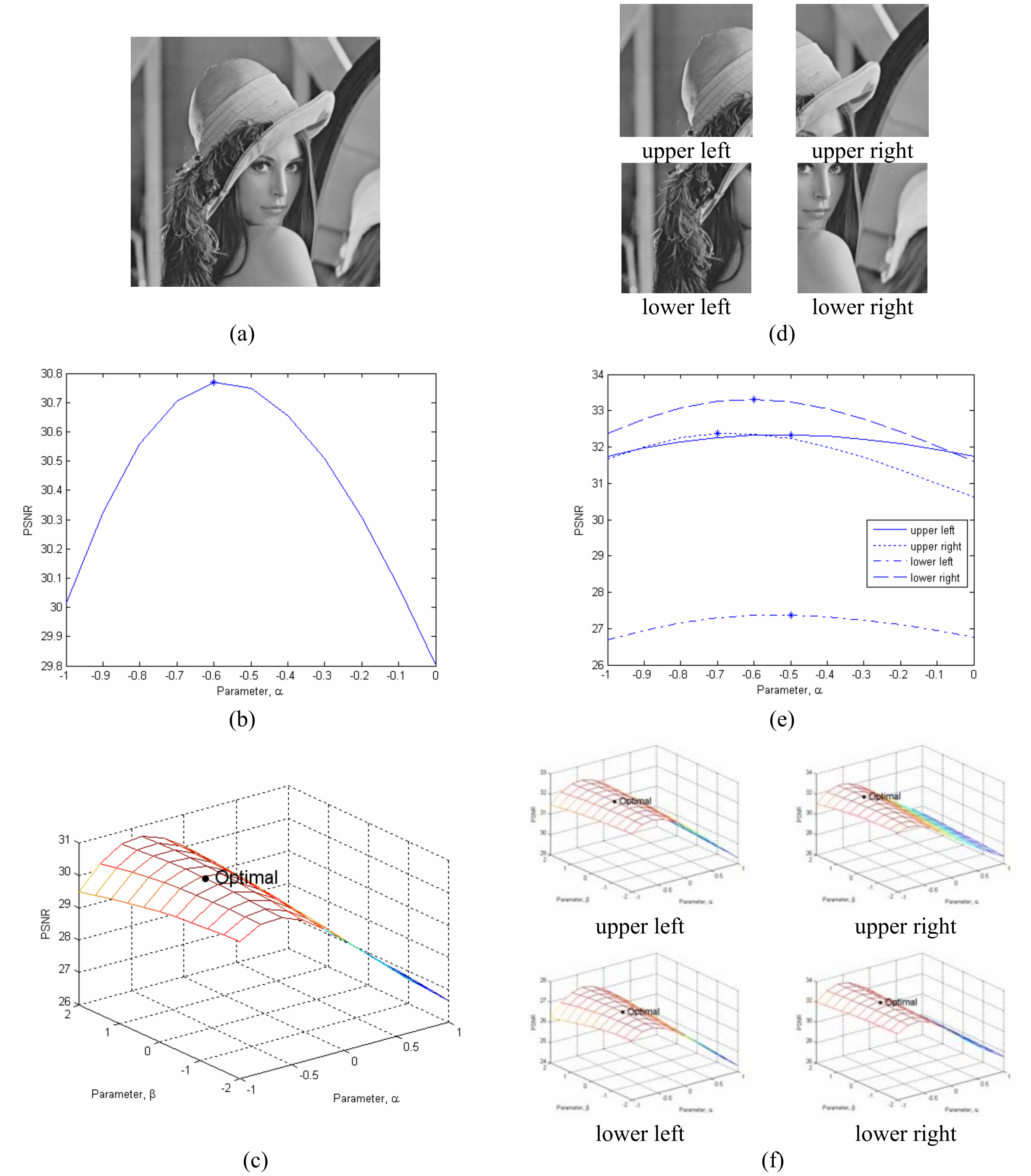


Fig. 3. Optimal parameters for "Lena". (a) original image; (b) PSNR versus α for separable cubic kernel; (c) PSNR versus α and β for nonseparable kernel; (d) original image is divided into 4 subimages; (e) PSNRs versus α for separable cubic kernel; (c) PSNRs versus α and β for nonseparable kernel.

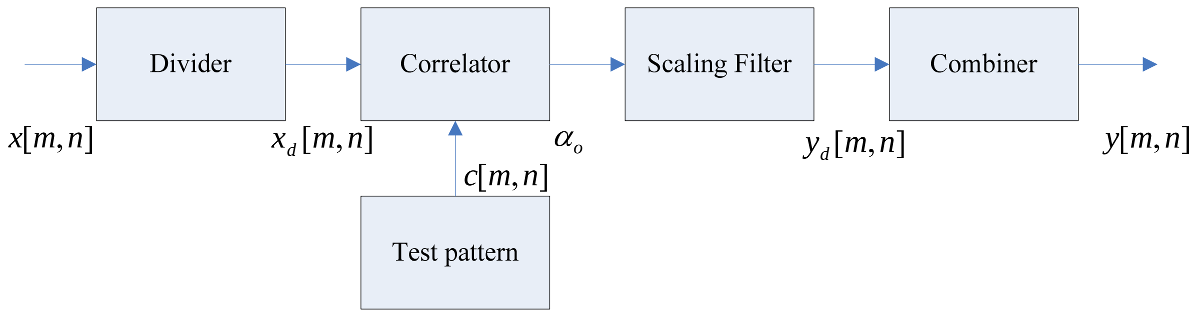
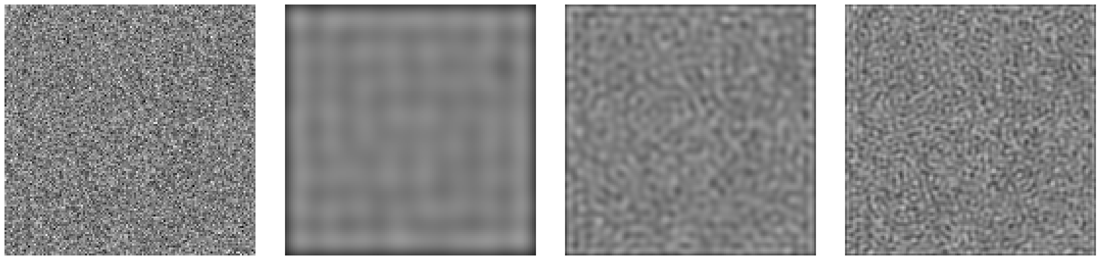
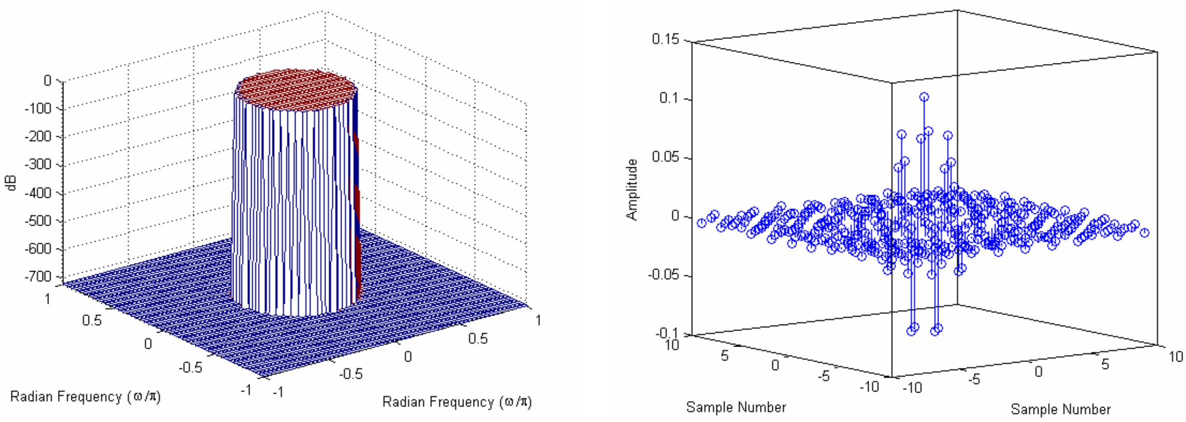


Fig. 4.　The flow chart of purposed methods.



(a) (b) (c) (d)

Fig. 5.　The test pattern whose mean is 128 and whose variance is 32. (a) noise image of independent identical Gaussian distribution; (b)-(d) images with correlated Gaussian distribution with cut frequencies $\pi/8$, $3\pi/8$, and $5\pi/8$ respectively.



(a) (b)

Fig. 6.　The low pass filter with cut frequency $3\pi/8$. (a) magnitude response; (b) impulse response.

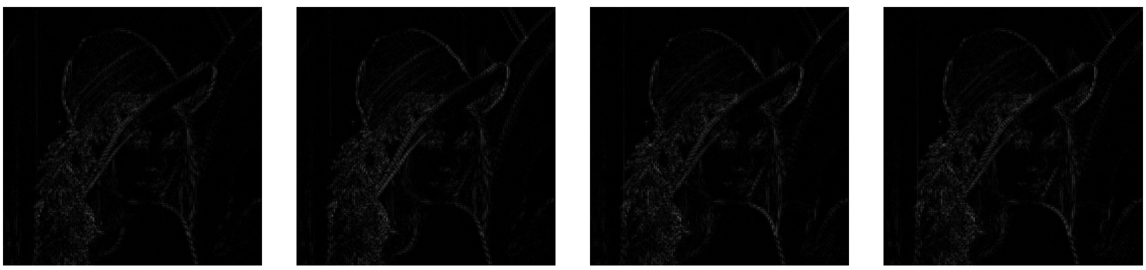
(a) (b) (c) (d)

(e) (f) (g) (h)

Fig. 7. Image used for experiments. (a) Baboon. (b) Lena. (c) Jet. (d) Moon. (e) Viewer. (f) Lake. (g) Room. (h) Peppers.

(a) (b) (c) (d)

Fig. 8. The 256 × 256 image "Lena" is scaled down by factor 2 × 2, and then scaled up by 2 × 2 by purposed method. (a-c) Scaled by separable kernel. The original image is divided into 4, 16 and 64 blocks, respectively. (d) Scaled by nonseparable kernel. The original image is divided into 64 blocks.



(a) (b) (c) (d)

Fig. 9. The absolute difference between original image and scaled one. (a-c) Scaled by separable kernel. The original image is divided into 4, 16 and 64 blocks, respectively. (d) Scaled by nonseparable kernel. The original image is divided into 64 blocks.

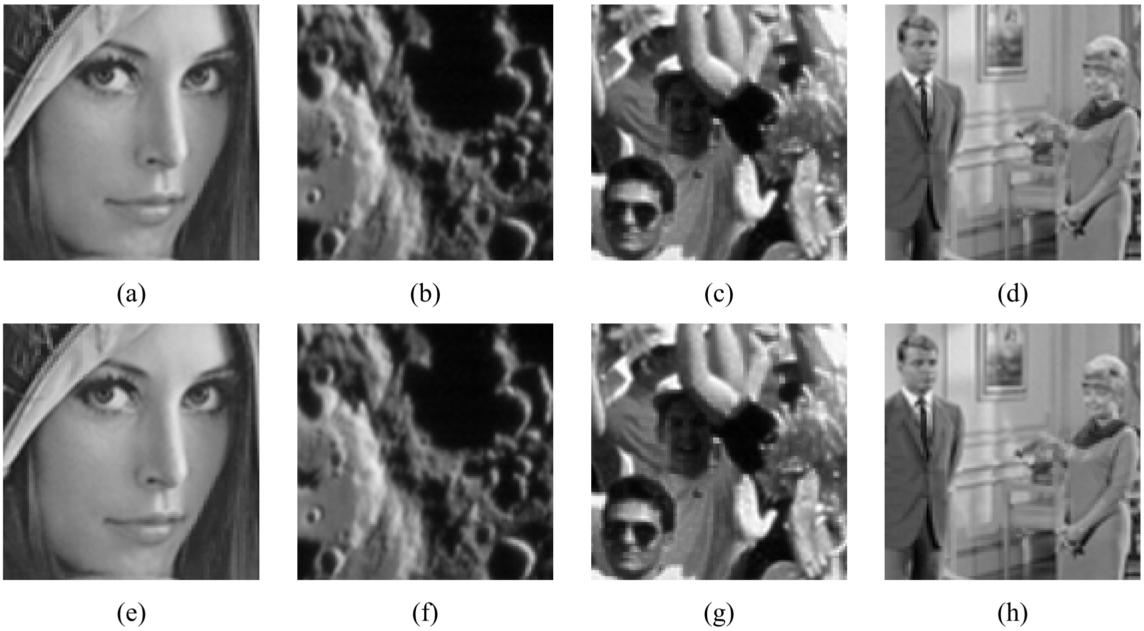


Fig. 10. Results of scaled up by factor 1.5 × 1.5. The original image is divided into 64 blocks (a-d) By separable kernel. (e-h) By nonseparable kernel.

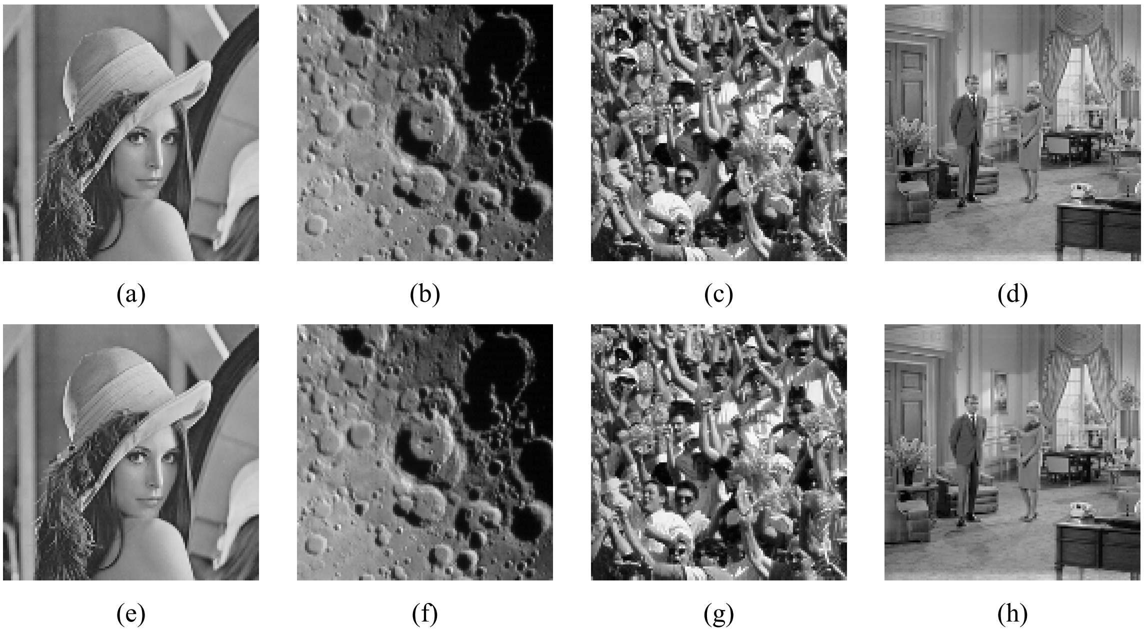


Fig. 11. Results of scaled up by factor 0.5 × 0.5. The original image is divided into 64 blocks (a-d) By separable kernel. (e-h) By nonseparable kernel.

TABLE I

PSNRs of image scaling using cubic kernel with fixed, separable and nonseparable kernels

Image	Cubic α = -0.5	Optimal Parameter	Parameters using Test Pattern			
			One			Two
		4/16/64 blocks	4 blocks	16 blocks	64 blocks	
Baboon	24.4	24.5	24.3	24.3	24.3	24.2
Lena	30.8	30.8	30.4	29.7	29.8	29.7
Jet	28.1	28.1	26.8	27.8	27.8	27.7
Moon	31.2	31.4	31.3	31.3	30.3	30.1
Viewer	25.0	25.1	25.0	24.6	24.1	24.1
Lake	27.2	27.2	27.2	26.7	26.8	26.7
Room	27.2	27.3	26.6	26.7	26.8	26.8
Peppers	31.6	31.7	30.9	30.4	31.0	30.9

相關性影像縮放利用可分離與不可分離遮罩

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摘要

立方迴旋積是影像重建最普遍的方式。立方迴旋積核心包含一個可調整參數值，一般來說固定為-0.5。但其參數值針對所有影像並非最佳化。在這篇論文裡，我們針對影像縮放，提出一種可選擇性適應參數值的立法內插。首先將原始影像切割成許多沒有重疊的子影像。接著依照每張子影像找出其最適合的參數值並且利用各自的參數去完成影像縮放。最後將所有子影像合併成完整影像。除了可分離核心之外，我們也可運用在二個可調整參數的非可分離核心上。實驗指出，如果可以找到最佳化參數值，其縮放影像必然比固定參數值擁用更高的品質。

關鍵詞：相關性縮放、可分離立法內插、不可分離內插。