# A Simple Edge Detection Method by Discrete Pascal Transformation

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# A Simple Edge Detection Method by Discrete Pascal Transformation

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### 摘要

本文提出一個簡單且有效的邊緣偵測法來偵測圖像或物體的邊，它是結合水平、垂直和對角方向同步檢測的技巧找出邊緣點。本方法所使用的技巧與動機是來自於一新穎的離散型巴斯卡轉換所具有的特殊性質。從性質推導觀察可看出，影像的邊緣點通常出現在巴斯卡轉換後系數間變號的位置，因此可做為檢測邊緣點的依據。最後，透過實驗結果可清楚看出此方法對於邊緣偵測之有效性。

關鍵詞：離散型巴斯卡轉換，邊緣偵測，同步。

### Abstract

In this article, a simple and effective edge detection method is presented and it locates edge points synchronously with horizontal, vertical, and diagonal. The innovation was based on a novel transformation which is named as discrete Pascal transform. By observation, edges usually locate on the position where the transformed coefficients alter signs. The experimental results demonstrate the efficiency of the proposed method on edges and fine details or unobvious edges especially.

Keywords: Discrete Pascal Transform, Edge Detection, Synchronously.

## 1. Introduction

Edges constitute significant portions of information contained in images. They can reduce vast complexity and retain important structures presented in images. Thus, it is important to extract these features from a scene for image processing, computer vision, and pattern recognition. Traditionally, edge detection mainly a two-stage process, edge enhancement followed by finding a feasible threshold. Most recipes for edge enhancement include the first order derivative detectors such as Prewitt [1], Roberts [2], and $\nabla G$ [3, 5] operators, and the second order derivative detectors such as Laplacian of the Gaussian and $\nabla^2 G$ operators [4, 6]. The simple enhancement techniques, Prewitt and Roberts operators, usually do well for ideal edges but perform poor for ramp or blurred edges. To sum up, the first stage seldom characterizes boundaries completely because of noise, breaks in the boundary from non-uniform illuminations, and other effects that introduce spurious intensity discontinuities due to the difficulty of choosing a proper threshold. The other computational approaches include Canny [3], Marr [4], Petrou [5], and Qian [6] model edges based on a set of criteria, such as detection, signal to noise ratio and good localization etc.

These approaches usually perform better than simple gradient techniques under complex real environment such as noisy images. The prices paid for the advantage are increased computational complexity and time.

Smoothing filters [3-4] are the most commonly used pre-processors before edge detection. They are usually tending to blur edges and causing unpleasant results as well as thick edges during edge detection phase. More reliable edge detectors were needed to overcome ramp or blurred edges caused by prior smoothing phase. For the reason, a non-threshold edge detector coupled with locator is purposed in sequels.

Discrete Pascal transform has succeeded in several applications of image processing includes digital filter design [11-12], three-dimensional object detection [13], digital water-marking [14], bump and edge detection [16]. Some of its interesting properties have been discussed in former researches [15-16]. In following contents, a new effective edge detection approach was innovated based on the novel transformation of discrete Pascal transform. Experimental results show that this approach has well ability in edge detection and do well on localization.

The organization of the work is as follows. In Section 2, the discrete Pascal transform is introduced firstly, and then the edge point locator derived from discrete Pascal transform is devised. The application of the present algorithm is given in Section 3. The utility is illustrated by a number of sample images in Section 4. Finally, the conclusion is given in Section 5.

## 2. Discrete Pascal Transform (DPT) [16]

### 2.1 Pascal Matrix and DPT

Recently, Maurice and Thomas [16] presented a new discrete transformation known as the discrete Pascal transform using a variation of the Pascal matrix for bump and edge detection. This new discrete transform related to the Pascal matrix [9-10] and possessed a lot of special properties such as the reversibility and identity etc. The definition of Pascal matrix is given as follows: Let $S$ be a symmetric square $N \times N$ matrix and coefficients of matrix $S$ is given as

$$S_{ij} = \binom{i+j}{i} = \binom{i}{j}\binom{j}{i} \text{ ........................ (1)}$$

for indexes $i, j = 0, 1, 2, ...., (N-1)$ , $S$ is called as Pascal matrix.

Given a sequence of data $y(n)$ with $n = 1, 2, \cdots, N$ , the one-dimensional discrete Pascal transform is given as follows:

$$Y = P \times y \text{ ............................................. (2)}$$

where $P$ is a $N \times N$ matrix and $y$ is a $N \times 1$ vector. The elements of the Pascal transform matrix $P$ are defined by

$$p_{ij} = \begin{cases} (-1)^j \binom{i}{j} & j \le i \\ 0 & otherwise \end{cases} \text{ ........ (3)}$$

Observing equation (1) and (3), coefficients show the relation of Pascal matrix and the discrete Pascal transform. The matrix $P$ can be obtained by placing the rows of Pascal’s triangle into matrix form and alternating the signs of the columns.

The two-dimensional discrete Pascal transform therefore was defined on two-dimensional case upon to one-dimensional DPT. Exact expression is given as:

$$Z = P \times z \times P^T \text{ ..................................... (4)}$$

where $z$ is a two-dimensional signal data with size of $N \times N$ .

### 2.2 A New Edge Locator by Discrete Pascal Transform

In this section, a non-threshold edge detector which relative to edge gradient will be innovated by the discrete Pascal transform. As in Maurice and Thomas [16] researches, they revealed that coefficients in the Pascal transform represent weighted difference in neighboring data values. A sharp change in the pixel values of an image will produce large coefficients in the transformed matrix. By scanning for large coefficients in the transformed matrix, we can determine whether or not a bump presents. However, how large threshold should be needed to detect bumps and edges dependent on different situations.

By equation (3), two trivial Pascal transform matrixes with size of $3\times 3$ and $4\times 4$ denoted as $P_3$ and $P_4$ are shown in Figure 1.

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & -3 & 3 & -1 \end{bmatrix}$$

**Figure 1:** Two different scale Pascal transform matrixes $P_3$ and $P_4$.

With above two transformation matrixes $P_3$ and $P_4$, an one-dimensional signal with edge point can be illustrated as one of the following five cases: $\begin{bmatrix} a & b & b \end{bmatrix}$, $\begin{bmatrix} a & a & b \end{bmatrix}$, $\begin{bmatrix} a & b & b & b \end{bmatrix}$, $\begin{bmatrix} a & a & b & b \end{bmatrix}$, and $\begin{bmatrix} a & a & a & b \end{bmatrix}$, where $a \neq b$. The one-dimensional Pascal transform of all five cases are given as:

Case 1: $P_3 \times \begin{bmatrix} a & b & b \end{bmatrix}^T = \begin{bmatrix} a & a-b & a-b \end{bmatrix}^T$ ............ (5)

Case 2: $P_3 \times \begin{bmatrix} a & a & b \end{bmatrix}^T = \begin{bmatrix} a & 0 & b-a \end{bmatrix}^T$ ............ (6)

Case 3: $P_4 \times \begin{bmatrix} a & b & b & b \end{bmatrix}^T = \begin{bmatrix} a & a-b & a-b & a-b \end{bmatrix}^T$ ............ (7)

Case 4: $P_4 \times \begin{bmatrix} a & a & b & b \end{bmatrix}^T = \begin{bmatrix} a & 0 & b-a & 2(b-a) \end{bmatrix}^T$ ............ (8)

Case 5: $P_4 \times \begin{bmatrix} a & a & a & b \end{bmatrix}^T = \begin{bmatrix} a & 0 & 0 & a-b \end{bmatrix}^T$ ............ (9)

It is clear that edge points locate on the position where the transformed coefficients alter signs. For cases 2 and 4, there exist an alternation of zero to negative number on transformed coefficients when $a > b$. Similarly, cases 1, 3, and 5 also present an change of number $a$ to negative number on transformed coefficients when $a < b$.

Let $f(x)$ be an one-dimensional signal and

$f_p^n(x) = [\, f(p) \quad f(p+1) \quad f(p+2) \quad \cdots \quad f(p+n-1)]$ ............ (10)

$f_{p-s}^n(x) = \left[ f(p-s) \quad \cdots \quad f(p) \quad f(p+1) \quad \cdots \quad f(p+n-s-1) \right]$ ............ (11)

be two sub signals of size $n$ started from position $p$ and $p-s$. A new edge locator for one-dimensional signal was innovated as that:

Finding positions $p$ where either $P_n \times f_p^n(x)$ or $P_n \times \bar{f}_p^n(x)$ exist a sign alternation, where $\bar{f}_p^n(x) = \sup(f_p^n(x)) - f_p^n(x)$ is the inverse signal of $f_p^n(x)$. However, this locator is sensitive to noise since that it can't differentiate between noise and edge. One of solutions is to shift position $p$ to $p-s$ to restrain. As $s = 0,1$ and $n = 3$, the resulting signals of shift in postion just like the mapping from Case 1 to Case 2. Similary, Case 3 to Case 5 mapping the situations of $s = 0,1,2$ and $n = 4$. By the observation, an edge point locates in the sub signal started from $p$ still belongs in another sub signal started from $p-s$. This is a dobule check for obtaining actual edge points and removing noisy points. That is obvious, the shift operation potentially coupled with the tolerance to noise. The edge locator can be pinpointed by following expression:

$$\mathop{find}_{e}\left(\prod_{\substack{s\in\Lambda \\ \Lambda\subset\Omega}} sign(P_n \times f_{p-s}^n(x) \vee \prod_{\substack{s\in\Lambda \\ \Lambda\subset\Omega}} sign(P_n \times \bar{f}_{p-s}^n(x)\right) \qquad (12)$$

where $\Omega = \{0,1,2,\cdots,\lfloor n/2 \rfloor\}$ and $\Lambda = \{0,1,2,\cdots,k\}$ be an ordered subset of $\Omega$. The greater of number $k$ we chose the larger of set $\Lambda$ obtained, and the designed edge locator is lower sensivity to noise.

## 3. Edge Detection by Discrete Pascal Transform

Most edge detectors, be the gradient-based approaches, require obtaining the edge gradient magnitude. Based on the gradient magnitude result, an approach is then designed to estimate the edge locations. In this section, we developed a non-threshold edge detection method relative to edge gradient on image processing. Intuitively, for a pixel $p$ in a two-dimensional image we applied the edge locator presented in above section according to the four directions shown in Figure 2: horizontal, vertical, forward diagonal, and backward diagonal. Once $p$ was located on one of the four directions, the pixel is regarded as an edge point. Notice that, it is worth to emphasize this novel detector is simple and full parallel by equation (12) and the four separable directions.

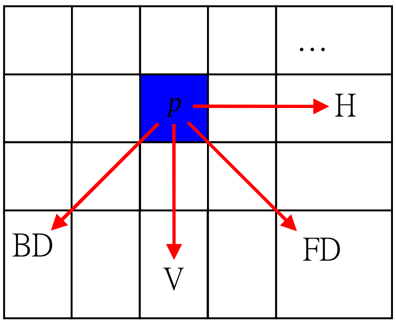



**Figure 2:** Four directions of the proposed edge detection algorithm, including horizontal, vertical, forward diagonal, and backward diagonal labeled as H, V, FD, and BD, respectively.

## 4. Experimental Results and Discussions

To explore the usability and demonstrate the efficiency of the proposed novel detector, we carry all experiments on below four gray-scale images shown in Figure 3. At first, we evaluate the efficiency of the simple locator proposed in section 2.2 on an artificial image depicted in the first picture on top row of Figure3. The resulted edge point image was shown in the first picture on top row of Figure 4 and the parameters we chose are that $n = 3$ and $k = 0$. It shows that the edge locator does well on ideal edges but not good on bump and noisy points as mentioned above.

It is worth noting, the proposed edge detector is a gradient-based approach. The gradient based methods give very little control over image noise and edge locations. Some kind of noise suppression method is needed. The Gaussian filter can serve the responsibility of smoothing for noise reduction. Since that the Gaussian filter smoothed the noisy image and also blurred the edge of image, thick edges usually were produced. Lots of edge detection methods utilize a post processing immediately after edge extraction to thin or skeletonize edges. There are many well-established thinning algorithms [17] (Louisa Lam, Seong-Whan Lee, and Ching Y. Wuen, 1992). For simplicity, the thinning algorithm described at the bottom of the first column and the top of the second column on page 879 was applied on the edge map detected by proposed method. Figure 4 shows the results of other five images, where the parameters we used are $n = 3$, $k = 1$, and a Gaussian variance $\sigma = 2$. Experimental results show that our method detected edges good on all sample images.

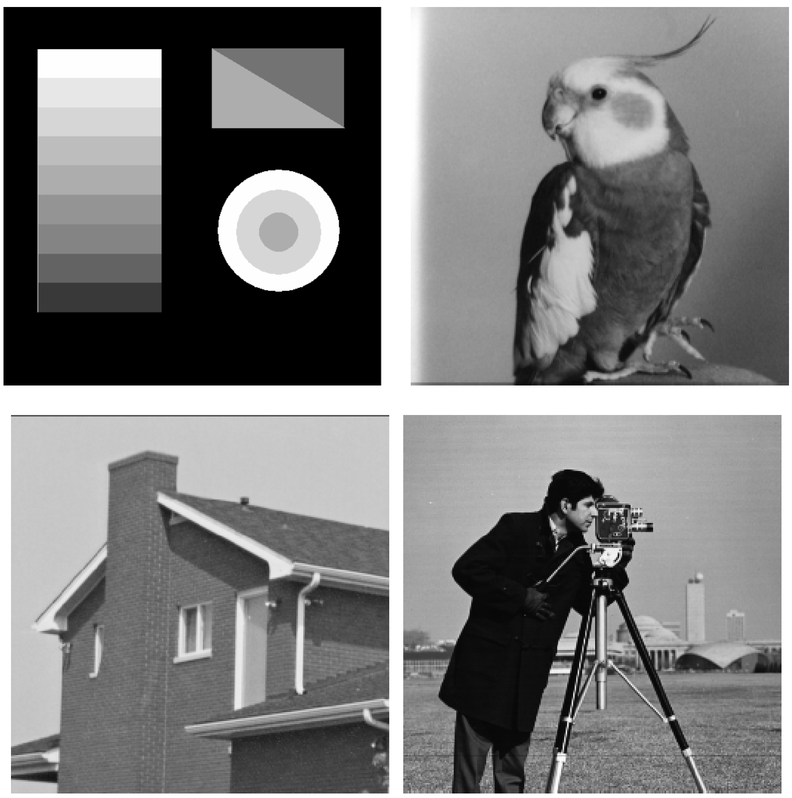


**Figure 3:** A set of sample images with size $256 \times 256$.

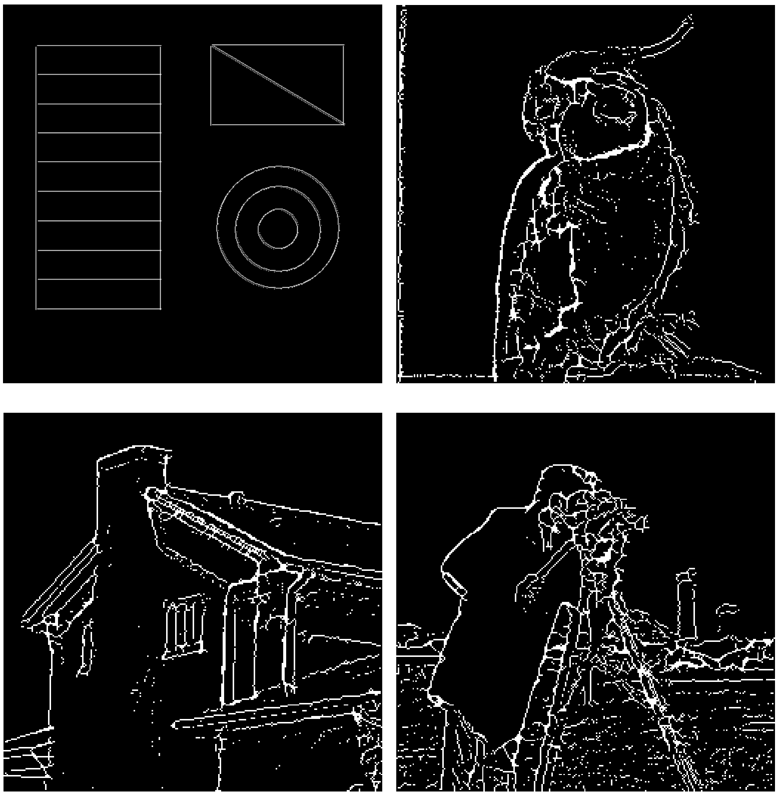


**Figure 4:** The resulted edge images of the Fig. 3 by proposed algorithm.

For comparison with other research efforts, we also explore the efficiencies of LOG (Laplacian of Gaussian) and Canny operators that are most researches used as a benchmark. Figure 5 shows the results of LOG (Laplacian of Gaussian) operator and Figure 6 shows the results of Canny operator. Clearly, the results obtained from LOG are the worst due to redundant pixels under testing images. Recently Heath [7-8] has been indicated that Canny operator [3] is a standard and good approximation but dependent on the selection parameters. Therefore, we explore the results of Canny shown in Figure 6. Figure 6(a) and (c) show the results of same as parameters $\sigma$=1.0, low threshold $t$ =0.3, and high threshold T=0.7. In Figure 6(b) and (c) provide well results for different images but Figure 6(a). As the parameter of low threshold $t$ decreases from 0.5 to 0.3 on the same image, however the result of Figure 6(a) is not so satisfied. In our methods, we always fixed on same parameters.

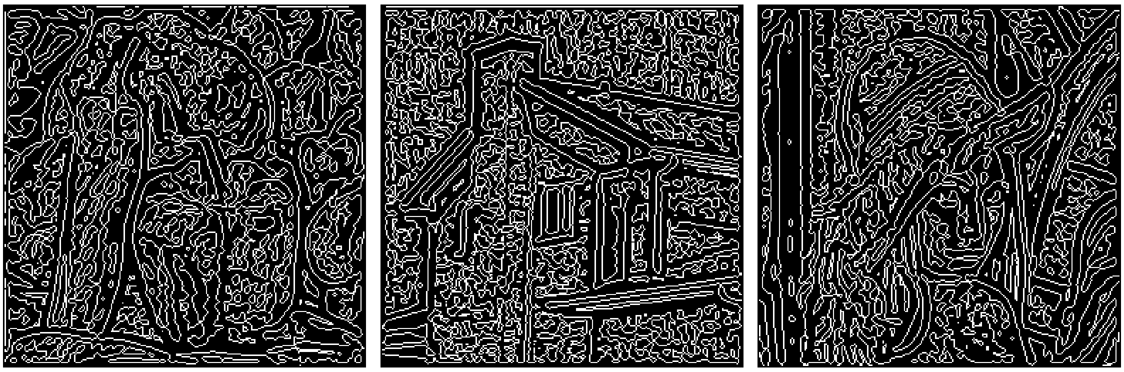
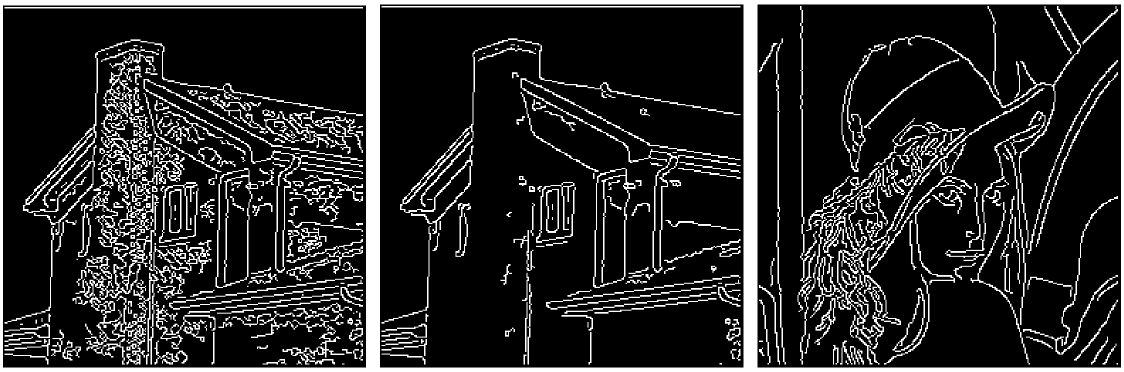


**Figure 5:** The results of LOG (Laplacian of Gaussian) with $\sigma$=2.0 on different images.



(a) σ=1.0, lt=0.3, ht=0.7; (b) σ=1.0, lt=0.5, ht=0.8; (c) σ=1.0, lt=0.3, ht=0.7.

**Figure 6:** The results of Canny operator with different parameters for sample images.

## 5. Conclusion and Discussions

In this paper, we developed a simple, effective and partial parallel edge location method using a novel discrete Pascal transform. The performance of the proposed method is compared with some of famous edge detection methods, including LOG and Canny detectors. The Canny operator can be optimized with respect to the filter length and time-frequency localization but need complex computation, whereas, our simple scheme exhibits more adroit on edges and fine-details.

Edge detection has an important role in computer vision and pattern recognition. Conventional edge detection techniques either often fail to produce satisfactory results for a broad variety of low-contrast images, or cannot be automatically applied to different images, because their parameters must be specified manually to produce a satisfactory result for a given image. No-threshold edge detection technique is more and more stringent. In this article we reveal the importance and propose a simple scheme for edge detection.

Besides, a good edge detection method also needs to have the abilities about noise holdout and finding edge direction. We are going to devote and design such good edge detectors to go on for more practical applications.

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